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Vibration Analysis Tire Treadband

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ABSTRACT

In this paper, the vibration of the tire treadband is discussed. For this purpose we consider the equations of motion of a cylindrical shell and then its natural frequencies are obtained numerically with the aid of MATLAB software. Then, the natural frequencies and mode shapes of the cylindrical shell shape are also obtained through Finite Element (FE) method using ANSYS. Results indicate acceptable values at low frequencies for the cylindrical shell. Moreover, the effect of variations of parameters such as pressure, temperature and the thickness of the cylindrical shell on natural frequencies are investigated.

KEYWORDS

Tire treadband , Cylindrical shell model, MATLAB, ANSYS

INTRODUCTION

The automotive industry is involved in a continuous endeavor to improve the noise and vibration characteristics of passenger vehicles. In particular tyres present a heavy influence on the overall vibrational behaviour of vehicles. Tyres are the interface between road and vehicle, so they are the links through which forces are exchanged, the presence of which can cause losses of energy, vibration and noise. From this point of view, the study of tyre vibrational behaviour becomes of fundamental importance [1]. Bolton and Kim [2-4] considered the stationary tire dispersion relations that characterize a tire's dynamics and its potential for sound radiation. They described both an experimental measurement procedure and a wave number decomposition technique for analyzing the radial vibration of a tire [2]. Moreover, analytical and numerical models of tire treadbands that were found to reproduce the significant features of measured tire dispersion relations were described [3]. Moreover, Jaesang and Seunghwan [5] simulated the behavior of the cords in the belt, and a wire cord finite element model was developed to accurately model the geometry of cords. The effects of some physical parameters such as the inflation pressure, tread pattern, thickness of belts and ply angles on the natural frequencies of tires were investigated [6]. In this article, results from the Finite Elements method are obtained and will be compared with those obtained using a cylindrical shell model. Finally, the effect of variations of parameters such as pressure, temperature and the thickness of the cylindrical shell on natural frequencies are investigated.

TIRE TREADBAND MODEL

Tyre tread can be treated as a thin finite cylindrical shell for adopting thin shell theory, because tyre tread thickness is less than 10% of the shell radius [7]. There are several shell theories in literature which differ based on their various assumptions regarding the order and form of the small terms threin [8]. The Donnel-Mushtari shell theory is by far the simplest and has been adopted in other tyre case studies for the simply supported condition [9].
Fig. 1 shows a cylindrical shell model of a tire treadband. In this model, the cylindrical shell was assumed to be the only elastic component. The cylindrical shell has thickness, radius R, and tire width L. The boundary conditions for the cylindrical shell were assumed to be simply supported.

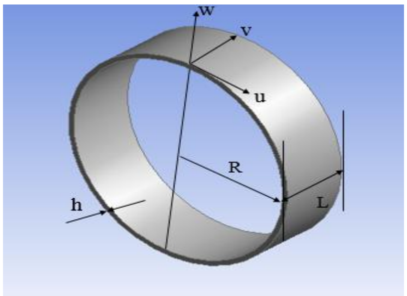

Fig. 1. Model of tire treadband: a circular cylindrical shell.

NATURAL VIBRATION

The free vibration of circular cylindrical shells with simply supported boundary conditions has been studied using different thin shell theories, including Donnell-Mushtari, Love-Timoshenko, Arnold-Warburton, Houghton-Johns, Flugge-Byrne-Lurye, Reissner-Naghdi-Berry, Sanders, Vlasov, Kennard-Simplified and Soedel [10].The

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equations of motion used for the thin circular cylindrical shell follow the Donnell-Mushtari theory [9]. The first step in the model of the tire structure is to solve for the eigenvalue problem of the self-adjoint Donnell-Mushtari operator. The displacement vector is defined as {u, v, w}T where (u) is the axial, (v) tangential, and (w) radial components as shown in Figure 2.1. For the simply-supported cylindrical shell of finite length, L, the harmonic displacement vector oscillating a frequency ω is given as

$$\{u_i\}=\begin{Bmatrix} U\cos(\lambda_m)\cos(n\theta)\cos(\omega t) \\ V\sin(\lambda_m)\sin(n\theta)\cos(\omega t) \\ W\sin(\lambda_m)\cos(n\theta)\cos(\omega t) \end{Bmatrix} \tag{2.1}$$

$i=1,2,3 \qquad n=0,1,2,... \qquad m=1,2,3,...$

where λ_m is:

$$\lambda_m = m\pi\frac{R}{L} \text{ for } m=1,2,3,...$$

In Equation (1), each pair of indices (m, n) define the modal pattern, i.e. m and n define the axial and azimuth variation of the mode shape. The displacement vector in Equation (1) defines the mode shapes. To find the mode natural frequencies, the Donnel–Mushtari operator is applied on the displacement as:

$$\begin{bmatrix} R^2\frac{\partial^2}{\partial x^2}+\frac{(1-\upsilon^2)}{2}\frac{\partial^2}{\partial\theta^2}-\rho\frac{(1-\upsilon^2)R^2}{E}\frac{\partial^2}{\partial t^2} & \frac{(1+\upsilon)R}{2}\frac{\partial^2}{\partial x\partial\theta} & \upsilon R\frac{\partial}{\partial x} \\ \frac{(1+\upsilon)R}{2}\frac{\partial^2}{\partial x\partial\theta} & \frac{\partial^2}{\partial\theta^2}+\frac{(1-\upsilon^2)R}{2}\frac{\partial^2}{\partial x^2}-\rho\frac{(1-\upsilon^2)R^2}{E}\frac{\partial^2}{\partial t^2} & \frac{\partial}{\partial\theta} \\ \upsilon R\frac{\partial}{\partial x} & \upsilon R\frac{\partial}{\partial x} & 1+K\nabla^4+\rho\frac{(1-\upsilon^2)R^2}{E}\frac{\partial^2}{\partial t^2} \end{bmatrix}.\begin{Bmatrix} u \\ v \\ w \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{2.2}$$

where ρ is the tire material density, E is the tire Young's Modulus, ν is the Poison ratio, and K is the nondimensional tire thickness parameter given by

$$K=\frac{h_{Shell}}{12R^2}$$

Replacing (1) into (2) leads to

$$\begin{bmatrix} -\lambda_m^2-\frac{(1-\upsilon^2)}{2}n^2+\Omega^2 & \frac{(1+\upsilon)}{2}n\lambda_m & \upsilon\lambda_m \\ \frac{(1+\upsilon)}{2}n\lambda_m & \frac{(1-\upsilon)}{2}\lambda_m^2-n^2+\Omega^2 & -n \\ -\upsilon\lambda_m & n & 1+K(\lambda_m^2+n^2)-\Omega^2 \end{bmatrix}.\begin{Bmatrix} U \\ V \\ W \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{2.3}$$

where Ω is called normalized frequency given as

$$\omega=(\frac{\Omega}{2\pi R})\sqrt{\frac{E}{\rho(1-\upsilon^2)}} \tag{2.4}$$

The eigenvalue problem in (3) will yield the shell natural frequencies and mode shapes. For each pair (m, n) defining a response pattern, the eigenvalue problem results in three natural frequencies which define three modes. In general, these modes are characterized by the dominance of one of the displacement vector component, i.e. longitudinally (i.e. $U_{mn}^{(j)} > W_{mn}^{(j)}$ and $U_{mn}^{(j)} > V_{mn}^{(j)}$), tangentially (i.e. $V_{mn}^{(j)} > U_{mn}^{(j)}$ and $V_{mn}^{(j)} > W_{mn}^{(j)}$), and radially (i.e. $W_{mn}^{(j)} > U_{mn}^{(j)}$ and $W_{mn}^{(j)} > V_{mn}^{(j)}$) dominated modes. The eigenvalues and eigenvectors are:

$$\Omega_{mnj} \text{ and } \begin{Bmatrix} U_{mn}^{(j)} \\ V_{mn}^{(j)} \\ W_{mn}^{(j)} \end{Bmatrix} \text{ for } (m,n) \quad j=1,2,3$$

By replacing the eigenvector from (5) into (1) leads to the eigenfunctions (or mode shapes)

$$\{\Phi_{mnj}\}=\begin{Bmatrix} U_{mn}^{(j)} \\ V_{mn}^{(j)} \\ W_{mn}^{(j)} \end{Bmatrix}\begin{Bmatrix} U_{mn}^{(j)}\cos(n\theta)\cos(\lambda_m\frac{x}{R}) \\ V_{mn}^{(j)}\sin(n\theta)\sin(\lambda_m\frac{x}{R}) \\ W_{mn}^{(j)}\cos(n\theta)\sin(\lambda_m\frac{x}{R}) \end{Bmatrix} \tag{2.5}$$

Characteristic radial modal patterns for a circular cylindrical shell supported at both ends by "shear diaphragms" are shown in Figures 2 and 3 (for 'vacuum' and 'inflated' cases). For instance, the $\Phi_{11}^{(1)}$ structural mode represents a half sine wave in the longitudinal direction and a complete cosine wave in the azimuth direction. The mode shapes with azimuthal index n=0 are referred to as "breathing" modes, while for n=1, they are noted as "bending" modes because the shell behaves similarly to a beam, i.e. no cross section deformation.

NUMERICAL SIMULATION

In order to illustrate the models developed in this chapter, a 195/65 R15 tire size was used for the simulations. The geometric parameters and material properties of the selected tire are given in Table 2.1. The material properties of the tire were provided by Tsihlas (Michelin North America) [9]. Table 2 shows the natural frequencies for the tire structure over the 200-400Hz frequency range. The dominant displacement component is also indicated in Tab. 1.

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Tab.1. Tire material and geometrical parameters

Parameter	Description
L=0.195 [m]	Tire Width
R =0.318 [m]	Middle radius
h_{Shell} =0.015 [m]	Thickness
7.5×10^{7} [N/m^{2}]	Young'sModulus
1350 [kg/m^{3}]	Material Density
υ =0.4	Poisson ratio

The natural frequencies calculated using Equation (4) are compared with those obtained from Finite Element method (ANSYS) for uncoupled case and listed in Table 3 for the range between 100–260 Hz. It is seen that for the lower radial mode numbers, Donnel-Mushtari shell theory gives acceptable results in comparison with finite element software. In order to match the mode numbers from the FE software with those from the analytical method, careful observation is needed because from Equation (4), a number of radial modes may have a lower resonance frequency than the lower radial mode number. In FE software however, the order of the modes are from low to high frequency. So in the FE result, the mode shapes has to be visually inspected to match the modes from Equation (2.4). For an actual tyre consists.

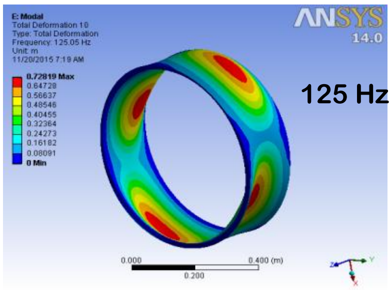

Fig.2. Tread natural frequency simulation for in vacuum cases at a frequency of 125 Hz using Ansys [11].

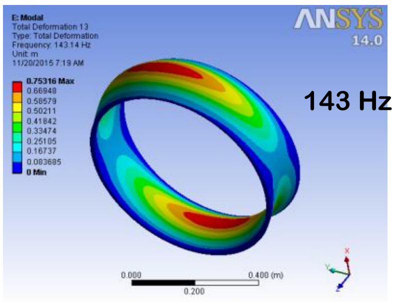

Fig.3. Tread natural frequency simulation for in vacuum cases at a frequency of 143 Hz using Ansys.

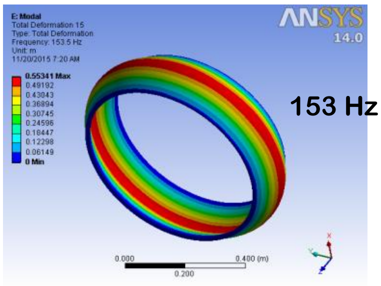

Fig.4. Tread natural frequency simulation for in vacuum cases at a frequency of 153 Hz using Ansys

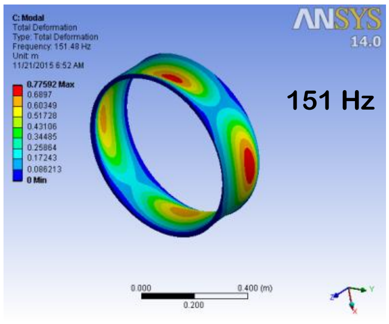

Fig.5. Tread natural frequency simulation for 'inflated' cases at a pressure of 210 kPa frequency of 151 Hz using Ansys .

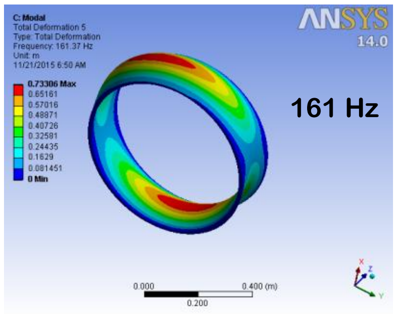

Fig.6. Tread natural frequency simulation for 'inflated' cases at a pressure of 210 kPa frequency of 161 Hz using Ansys .

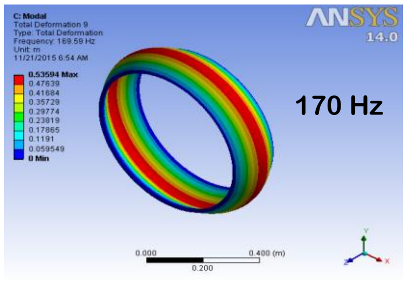

Fig.7. Tread natural frequency simulation for 'inflated' cases at a pressure of 210 kPa frequency of 170 Hz using Ansys

Journal of Automotive and Applied Mechanics

Volume 4,Issue 1,2016
http://www.scijour.com/jaam

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Tab.2. Tire structure natural frequencies between 200 and 400 [Hz]

m	n	Frequency [Hz]	Mode Type
2	0	218.40	Radial
2	1	219.24	Radial
1	10	221.69	Radial
2	2	221.94	Radial
2	3	226.92	Radial
2	4	234.76	Radial
2	5	246.04	Radial
1	11	257.98	Radial
2	6	261.18	Radial
2	7	280.42	Radial
1	12	297.96	Radial
2	8	303.82	Radial
2	9	331.37	Radial
1	13	341.59	Radial
2	10	362.96	Radial
1	1	369.15	Tangential
1	2	392.3	Tangential
2	11	398.48	Radial

Tab.3. Tread natural frequencies for 100- 260 [Hz]

Mode (m, n)	Ansys (Hz)	Analytical (Hz)
(1, 4)	104.07	102.56
(1, 3)	107.52	105.72
(1, 5)	108.55	106.87
(1, 2)	118.17	113.94
(1, 6)	121.51	118.75
(1, 1)	134.78	122.53
(1, 7)	139.76	137.15
(1, 8)	150.71	160.91
(1, 9)	156.30	189.25
(2, 0)	181.88	218.4
(2, 1)	193.27	219.24
(1, 10)	202.6	221.69
(2, 2)	210.97	221.94
(2, 3)	214.34	226.92
(2, 4)	228.02	234.76
(2, 5)	242.42	246.04
(1, 11)	243.17	257.98

Effective parameters

A) Pressure

Fig. 4 depicts the effect of different inflation pressures on the natural frequencies of the tire treadband. The inflation pressure directly influences the stiffness of the tire. As inflation pressure increases, the stiffness of the tire also increases and therefore the natural frequencies increase.

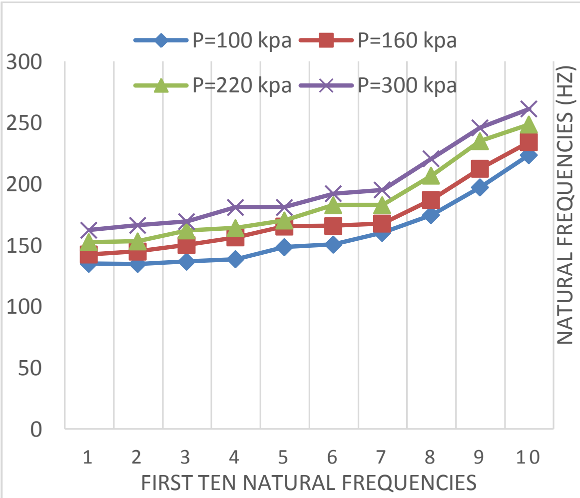

Fig.8. Variation of inflation pressure and natural Frequency.

B) Thickness

It is seen that the natural frequencies are not significantly affected by variation in the thickness of different tire treadband.

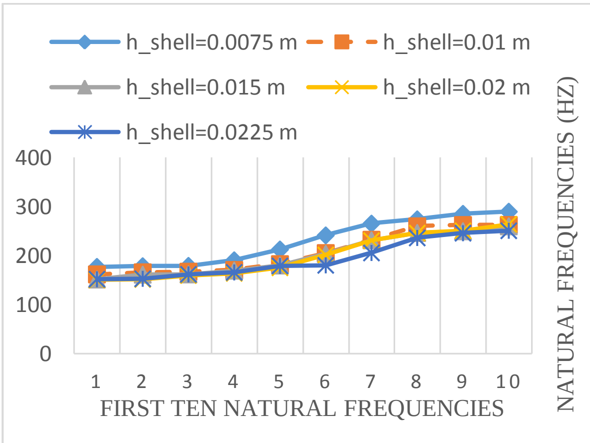

Fig.9. Variation of thickness and natural Frequency.

C) Temperatuer

Evaluation of natural frequencies in the temperature range of 15 to 40 ° C shows that the impact of temperature on the natural frequencies is negligible.

Conclusion

Natural frequencies calculated through analytical models were compared with those obtained from Finite Element method. It is seen that for low radial mode numbers, Donnel-Mushtari shell theory gives acceptable results, comparable with finite element software simulations. In order to match the mode numbers from the FE software with those from the analytical method, careful observation is needed, because a number of radial modes calculated using Equation (4) may have a lower resonance frequency than the lower radial mode number. Moreover, it was shown that inflation pressure directly influences the stiffness of the tire. As inflation pressure increases, the stiffness of the tire also increases and therefore the natural frequencies increase.

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