Comparison of Forecasted Accuracy Based on GARCH and State Space Models by using The Monte Carlo Simulation Method For Two indexes (The TEPIX and The S&P500)

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INTRODUCTION
Prediction of future events plays a major role in decision-making process. Thus, it is of great importance for many organizations and institutions. Furthermore, prediction of economic variables has a significant impact on the government policies. Therefore, accuracy level in forecasting these variables is considered among the secrets of success for such policies, regardless of correctness and appropriateness of the policies to the current situation.

The present challenges in forecasting time-series variables are mainly influenced by a development in the methods and instruments provided for prediction and consecutively, the significance of forecasting the time-series variables has generated diversity and extensity in such instruments. Hence, the comparison of the predictive power of different methods based on various criteria requires continuous and extensive investigation and researches.

Stock market indexes and especially overall index are considered as the most important criteria to evaluate investment performance in the stock market and even non-financial investments. The indexes are useful instruments to trace market trends. Understanding the index and its changes can help investors make more appropriate investment decisions. Therefore, the index efficiency is viewed as a part of stock market efficiency.

Certainly, it should be noted that appropriateness and predictive power of these models are different depending on the nature of available data. To have a deeper understanding of changes in prices, asset returns and risk management, it is necessary to identify the nature and behavior of uncertainty element. It is required for those individuals going to forecast the prices particularly in the financial markets to know this issue.
In this study, the TEPIX index and S&P500 index have been used to investigate the nature of these indicators as well as to investigate the predictive power of GARCH and State Space models using Monte Carlo simulation in different time horizons and using the RMSE criterion to consider these two important indexes. Same size and almost the same time period were used for the samples used in this study. These two indexes have been selected fully informed in the related time period, so that in the end, we cite to the comparisons done and specifying model or the number of samples do not cause disturbance in the comparisons.

In Section 2, we will briefly review the research literature. In section 3, the data will be describes, the models will be analyzed and the main results of the paper will be presented. In Section 4, Monte Carlo simulation technique for GARCH and State Space models and stipulated form of the two TEPIX and S&P500 are presented. Finally, in Section 5, the final results of the comparison are reported.

Overview of research literature

The ability to predict by statistical and econometric tools in the financial markets is one of the important issues many researchers are studying about them. There are many econometric methods for forecasting in the financial market, but the issue that which one is more appropriate needs to be investigated. In the past three decades, researchers increasingly have paid attention to prediction in different types of financial and non-financial markets using robust econometric models and various statistical and mathematical techniques; but for every particular market, the appropriate model should be selected to predict future market conditions. In this regard, many economists and researchers have compared different models for different markets and their achievements over past two decades (1989 to 2014) will be briefly overviewed in the following part.

Akgiray (1989) [1], Pagan and Schwet (1989) [14] used the data of American Stock and showed that GARCH (Generalized Autoregressive conditional Heteroscedasticity) models show better performance than other competing models. Dimson and Marsh (1990) [5] showed that in the UK stock market, thrifter models such as smooth and simple regression models perform better than less thrifter models; although GARCH models are not considered in the collection of competing models. Using Japanese and Singaporean market data sets, Tse (1991) [16] and Tse and Tung (1992) [17] found that the exponential weighted moving average models provide more accurate forecasts than the GARCH model. Akgiray (1989) [1] and Brailsford and Faff (1996) [3] found that the GARCH (1,1) model has a lower RMSE in forecasting volatility of S&P500 index over the long term but the historical variance model has a prediction for interest rate and foreign exchange markets and in another study in the same year, they found that GARCH models have relatively superiority than simpler models of forecasting for forecasting the volatility of daily data of Australian stock index. Jorion (1995) [12] reached the same conclusion about the foreign exchange market, while these studies have shown that the predictive ability of GARCH models is low. Andersen and Bollerslev (1997) [2] found that the predictive power increases by increasing the sample frequency, such as daily data and GARCH family models will show better performance.

West and Cho (1995) [18] found that volatility in foreign exchange markets is unpredictable for more than 5 days. Their findings were consistent with the findings of Andersen and Bollerslev (1997) [2], Brooks and Lee (1997) [4] used different types of ARCH/GARCH models to investigate data and Australian financial analysis. The result was that simpler models like ARCH (1) provide better fitness for data.

KhurramSaleem Malik (2000) [15] compared seven AR (Autoregressive), ARMA (Autoregressive moving average), ARCH, PPP (Purchasing Power Parity), DB (Dornbusch Frankel sticky price monetary), Decomposition of time series, and combined forecast models in terms of their forecasting accuracy and used monthly data over the period of January 2000 to June 2010 to predict the exchange rate in Pakistan and finally between these models, ARCH model was selected as the most accurate model. Durbin J. (2004) [6] held an extensive public review on State Space Approach analysis for Time Series by introducing State Space Linear Gaussian model and Kalman filter and he also described moderators. Javier Garcia-Cicco and Roque Montero (2011) [8] conducted a research on the prediction of the price of copper in the London market; the data used relates to the January 1975 to January 2010. AR (2) and GARCH (1,1) models were selected as the appropriate models and were compared with Markov switching model; the result was that as compared with data output, Markov switching model is better as compared to the AR model at all horizons. Lars FivaSkarbovik (2013) [10] compared three univariate time series models of AR, ARIMA and ETS (Exponential Smoothing State Space) in order to find the most accurate model for predicting house prices in Norway using data from the period of April 2013 to March 2014. After analysis, the ETS model was introduced as the most accurate model among the selected models.

Review of the literature indicates that the ARCH family models have been widely used and compared for prediction of various time series and researchers have become more interested in the use of State Space models in the debate of prediction in recent years; however, they were not very active in comparing this model with others. Also, the literature suggests that, in the time series of financial markets, there is the impact of volatility that represents efficiency of ARCH family for forecasting. According to the research literature and because of the paucity of comparative studies of different models in different time horizons and the importance of more accurate forecasting of the S&P500 and TEPIX indexes for financial markets and policymakers, activists and investors of these markets, we decided to conduct a research in this field and based on the research literature, the ARCH / GARCH model, due to its efficiency in forecasting and investigating financial variables, and State Space model, due to its unique features in the identification of the unknown parameters, were investigated and tested using Monte Carlo simulation method in order to maximize the accuracy of the prediction to identify the most
accurate model for forecasting the S&P500 and TEPIX indexes and to identify the nature of the behavior of these indexes.

**Data description and analysis of models**

In this section, the selected model for this study will be made and forecasting and simulation using these models and using the Monte Carlo method is discussed. It should be noted that to demonstrate the procedure, the long time forecasts and comparison with the out of sample data is used and the same steps are used for any given time horizons and for general comparison with in-sample data; so, in order to avoid repetition, the charts of other time horizons and the chart of the comparison based on the in-sample data are not given and only the final results of the mid time and short time horizons and the general comparison with the in-sample data have been reported.

### A) Data Analysis

The present research focuses on the investigation and forecasting the S&P500 and TEPIX indexes. For this purpose, the daily data of 21 January 2011 to 19 February 2014 were used as the in-sample data and the data of 20 February 2014 to 21 May 2014 were used as out of sample data for TEPIX index and the daily data of 18 March 2011 to 26 February 2014 were used as the in-sample data and the data of 27 February 2014 to 21 May 2014 were used as out of sample data for S&P500 index.

The samples are identical in terms of the sample size and are almost at the same time, in addition, the Bai and Perron test results showed the absence of structural breaks in the selected time series. The in-sample data are equal to 740 observations and the size of the out of sample data for the three time horizons is 59 observations, equal to 3 months (long time), 20 observations, equal to 1 month (for the mid time) and 5 observations, equal to 1 week (for short time). (Fig. 1 and Fig. 2)

### B) Investigating the Unit Root Test (ADF)

Augmented Dickey-Fuller test and the critical values of MacKinnon and three criteria of Schwarz, Akaike and Hannan-Quinn were used to evaluate the stationary of the time series. According to the tests conducted, the results of stationary tests indicate that the first difference of indexes' logarithms must be used instead of a series of indexes or logarithmic series of indexes in the modeling.

### C) Auto Regressive Conditional Heteroscedasticity and Stipulating GARCH models

As Engle (1982) has suggested, autoregressive conditional Heteroscedasticity of Lagrange multiplier has been used to detect the presence or absence of ARCH or GARCH model. According to this test, we concluded that models of ARCH family can be used for TEPIX and S&P500 indexes and this model doesn't have serial correlation. Thus, the Akaike and Schwarz and Hannan-Quinn criteria are used to stipulate GARCH models and both for the S&P500 and TEPIX indexes, the best model from ARCH family is the ARCH (1) or GARCH (0,1) model and all the estimated coefficients in the model are significant at the 99% level. Finally, the equation of GARCH (0, 1) model for TEPIX index (Eq 1) and the S&P500 (Eq 2) index is written as follows:

\[
\text{GARCH} = 4.5e^{-05} + 0.39 \varepsilon_t^2 (1)
\]

\[
\text{GARCH} = 9.2e^{-05} + 0.13 \varepsilon_t^2 (2)
\]

### D) Estimation of State Space Models for the S&P500 and TEPIX indexes and their stipulation

According to the Akaike and Schwarz Bayesian values, the best order for the State Space Model for both of the indexes is AR (2) and by placing the estimated coefficients in it, the following state and observation equations to forecast the TEPIX index (Eq 3 and Eq 4) and the S&P500 index (Eq 5 and Eq 6) are obtained.
State equations for the TEPIX index:
\[ x_{1(t)} = -(1.71)x_{1(t-1)} + (0.96)x_{2(t-1)} + (3.33e-07)u_{1(t)} \]
\[ x_{2(t)} = x_{1(t-1)} \]  \hspace{1cm} (3)

Observation equation for the TEPIX index:
\[ y_{1(t)} = x_{1(t)} + (9.87e-03)e_{1(t)} \]  \hspace{1cm} (4)

State equations for the S&P500 index:
\[ x_{1(t)} = -(0.47)x_{1(t-1)} + (0.34)x_{2(t-1)} + (2.63e-03)u_{1(t)} \]
\[ x_{2(t)} = x_{1(t-1)} \]  \hspace{1cm} (5)

Observation equation for the S&P500 index:
\[ y_{1(t)} = x_{1(t)} + (9.72e-03)e_{1(t)} \]  \hspace{1cm} (6)

**Simulation using Monte Carlo technique for stipulated GARCH and State Space Models related to two indexes of TEPIX and S&P500**

In Fig.3, the out of sample data and forecasts based on the ARCH model using Monte Carlo methods for TEPIX index and S&P500 index are displayed. In Fig.4 and Fig.5, the out of sample data and forecasts based on State Space Model for TEPIX index and S&P500 index can be seen. As is shown, the out of sample data does not go out of the forecasted area, this, in turn can lead us to conclude that the GARCH and State Space models have been stipulated very well and predictive parameters were stable.

**Comparison and Conclusion**

According to the results in Tab.1, GARCH model showed better performance in all the time horizons and even in comparison with the in-sample data and this is because of the presence of volatility in financial markets related to these two indicators that even in a period of 5 days, this volatility can cause GARCH model to be selected as the most appropriate model.

Using Monte Carlo simulation method gave us the power to have more accurate forecasts and thus a lower RMSE. Finance professionals can have confidence that they can have more accurate forecasts using econometric Science and employing a variety of different economic, physical,
mathematical and statistical methods, resulting in better decisions and having better policies. If the appropriate model is found, more appropriate decisions can be made even in volatility. Based on the findings of this study, stockbrokers, investors and financial analysts in the market are suggested to employ more advanced models and modern econometric - financial methods and professionals in this area, in addition to traditional approaches for their analysis.

Tab.1. The mean of RMSE for each model at different time horizons for the two TEPIX and S&P500 indexes

<table>
<thead>
<tr>
<th>Model</th>
<th>TEPIX Index</th>
<th>S&amp;P500 Index</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Long Time</td>
<td>Mean</td>
</tr>
<tr>
<td>GARCH</td>
<td>0.0109</td>
<td>0.0122</td>
</tr>
<tr>
<td>State Space</td>
<td>0.0120</td>
<td>0.0124</td>
</tr>
<tr>
<td>Mid Time Mean</td>
<td>0.0105</td>
<td>0.0119</td>
</tr>
<tr>
<td>GARCH</td>
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<tr>
<td>State Space</td>
<td>0.0092</td>
<td>0.0127</td>
</tr>
<tr>
<td>Short Time Mean</td>
<td>0.0107</td>
<td>0.0125</td>
</tr>
<tr>
<td>GARCH</td>
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<td>0.0147</td>
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<tr>
<td>State Space</td>
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<td>0.0148</td>
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</tbody>
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REFERENCES


