Journal of Automotive and Applied Mechanics
Volume 4,Issue 1,2016
http://www.scijour.com/jaam
SCIJOUR

Static Analysis of Sandwich Beams with FGM Core Resting on Elastic Foundation

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ABSTRACT

In this paper, Static analysis of sandwich beams with functionally graded materials (FGM) core resting on Winkler foundation are investigated using second order shear deformation beam theory (SBT). Material properties of the FGM layer change in the thickness direction according to power-law distributions. The governing equations and the related boundary conditions are derived using the principal of the minimum total potential energy. The Navier-type solution is used for simply-supported boundary conditions, and exact formulas are proposed for static analysis. The method is validated by comparing numerical results with the results obtained in the literature. Good agreement is observed. Numerical results are presented to investigate the influences the different material distributions and foundation stiffness on the static behavior of sandwich beams with FGM core.

KEYWORD

Beam , Functionally Graded, Static, Winkler

INTRODUCTION

A new class of composite materials, called functionally graded materials (FGMs), is considered in this paper. The potential uses of FGMs in engineering applications include aerospace structures, engine combustion chambers, fusion energy devices, engine parts and other engineering structures. In recent years, the static and dynamic analyses of functionally graded (FG) beams have increasingly attracted many researchers. Akbas [1] investigated static analysis of an edge cracked FGM beam resting on Winkler foundation by using finite element method. The dynamic response of FG beams with an open edge crack resting on an elastic foundation subjected to a transverse load moving at a constant speed is studied by Yan et al. [2]. Arefi [3] studied the nonlinear responses of a FG beam resting on a nonlinear foundation. Duy et al. [4] examined free vibration of FG beams on an elastic foundation and spring supports. Shen and Wang [5] investigated the large amplitude vibration, nonlinear bending and thermal post- buckling of FG beams resting on an elastic foundation in thermal environments. Li and Shao [6] studied nonlinear bending problem of FG cantilever beams resting on a Winkler elastic foundation under distributed load are discussed. Babilio [7] examined the nonlinear dynamics of axially graded beams rest on a linear viscoelastic foundation under axial time-dependent excitation is studied. Gan and Kien [8] studied a finite element procedure for the large deflection analysis of FG beams resting on a two-parameter elastic foundation.

Ying et al. [9] presented exact solutions for bending and free vibration of FG beams resting on a Winkler-Pasternak elastic foundation based on the two-dimensional theory of elasticity. Omidi et al. [10] studied the dynamic stability of simple supported FG beams resting on a continuous elastic foundation. Ahuja and Duffield [11] have investigated both theoretically and experimentally about the dynamic stability of beams having variable cross sections and resting on elastic foundation. The effect of elastic foundation is found to have decreased the width of the instability regions and the amplitude of parametric response.

Engel [12] has investigated the dynamic stability of bars on elastic foundation with damping. It is found that the critical mode becomes a higher mode instead of fundamental mode when the foundation parameter exceeds a certain value. Lee and Yang [13] and Matsunaga [14] have investigated the dynamic behavior of Timoshenko beams resting on elastic foundations The performance of graded finite element has been compared with conventional homogeneous finite elements by Santare and Lambros [15] to be used preferably in applications like failure analysis or crack path detection problems wherein the local stress values are of critical importance. Chen and Chan [16] have developed integral finite elements to estimate the dynamic characteristics of elastic-viscoelastic composite (EVC) structures. Paulino and Jin [17] have made an attempt to show that the correspondence principle can be applied to the study of visco-elastic functionally graded material (FGM) under the assumption that the relaxation moduli for shear

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Journal of Automotive and Applied Mechanics
Volume 4,Issue 1,2016
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dilation are separable functions in space and time. Chakraborty et al [18] have developed a beam finite element to study the thermo elastic behavior of functionally graded beam.

FUNCTIONALLY GRADED MATERIALS

There are some models in the literature that express the variation of material properties in FGMs. The most commonly used of these models is the power law distribution of the volume fraction. According to this model, the material property gradation through the thickness of beam is assumed to be the following form:

$$E = E(z) = (E_c - E_m)\left(\frac{z}{h_f} + \frac{1}{2}\right)^n + E_m \qquad (1)$$

Here E denote the modulus of elasticity of FGM layer, while these parameters come with subscript m or c represent the material properties for metal and ceramic respectively. For FGM layer, the thickness coordinate variable is presented by z while, $h_m \leq z \leq h_f + h_m$ where h_f is the total thickness of the FGM layer as shown Fig. 1.

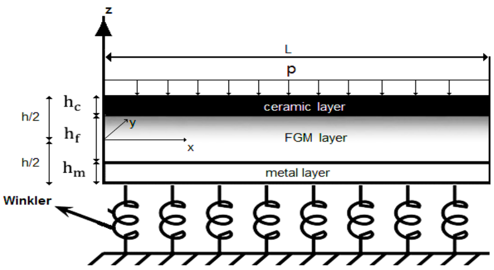

Fig. 1. Geometry and coordinates of sandwich beam resting on Winkler foundation.

THEORY AND FORMULATIONS

Consider a sandwich beam of length L, width b, thickness h, subjected to uniform distributed load P, resting on Winkler foundation with spring constant (K_w) as shown in Fig. 1.

DISPLACEMENT FIELD AND STRAINS

According to the coordinate system (x, y, z) shown in Fig. 1, based on second order shear deformation beam theory (SBT), the axial and the transverse displacement field are expressed as

$$u(x,z) = u_0(x) + z\Phi_1(x) + z^2\Phi_2(x) \qquad (2.1)$$
$$V(x,z) = 0 \qquad (2.2)$$
$$w(x,z) = w_0(x) \qquad (2.3)$$

Where U, V, W are x, y, z components of the displacements, respectively. Φ_1 and Φ_2 are the total bending rotation of the cross-section at any points on the neutral axis. Also, u_0, w_0 are the axial and the transverse displacements in the mid-plane. The strain-displacement equations of the linear strain are given by

$$\varepsilon_{11} = \varepsilon_{11}{}^{(0)} + z k_{11} + z^2 k_{11}{}^{(0)} \qquad (3.1)$$
$$\gamma_{13} = \gamma_{13}{}^{(0)} + z\,\gamma_{13}{}^{(1)} \qquad (3.2)$$

Where

$$\varepsilon_{11}{}^{(0)} = \frac{\partial u_0}{\partial x}, \qquad k_{11} = \frac{\partial \Phi_1}{\partial x}, \qquad k_{11}{}^{(0)} = \frac{\partial \Phi_2}{\partial x}$$
$$\gamma_{13}{}^{(0)} = \Phi_1 + \frac{\partial w_0}{\partial x}, \qquad \gamma_{13} = 2\Phi_2$$

STRESS-STRAIN RELATIONS

The stress-strain relations are given by

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{13}\end{bmatrix} = \begin{bmatrix}c_{11} & 0\\ 0 & c_{55}\end{bmatrix}\begin{bmatrix}\varepsilon_{11}\\ \gamma_{13}\end{bmatrix} \qquad (4)$$

Where

$$c_{11} = \frac{E}{(1-\nu^2)}, \qquad c_{55} = \frac{E}{2(1+\nu)}$$

EQUATIONS OF MOTION

The Hamilton's principle for an elastic body can be represented

$$\int_{t_1}^{t_2} \delta\left(U_p + U_f + V\right)dt = 0 \qquad (5)$$

Hamilton's principle, equation (5), by considering SBT, equations (2), yields the complete form of the equilibrium equations:

$$\delta u_0: \quad \frac{\partial N_{11}}{\partial x} = 0 \qquad (6.1)$$
$$\delta \Phi_1: \quad \frac{\partial M_{11}}{\partial x} + \Phi_{13} = 0 \qquad (6.2)$$
$$\delta \Phi_2: \quad \frac{\partial L_{11}}{\partial x} + R_{13} = 0 \qquad (6.3)$$
$$\delta w_0: \quad \frac{\partial Q_{13}}{\partial x} - K_w w_0 + P = 0 \qquad (6.4)$$

Where N, M, L, R and Q are the stress resultants. These parameters can be present by

$$\begin{Bmatrix}N_{11}\\ M_{11}\\ L_{11}\end{Bmatrix} = \int_{-h/2}^{h/2} \sigma_{11}\begin{Bmatrix}1\\ z\\ z^2\end{Bmatrix} dz \qquad (7.1)$$
$$\begin{Bmatrix}Q_{13}\\ R_{13}\end{Bmatrix} = \int_{-h/2}^{h/2} \sigma_{13}\begin{Bmatrix}1\\ z\end{Bmatrix} dz \qquad (7.2)$$

The stress resultants can be expressed in terms of the strains as

$$N_{xx} = A_{11}\varepsilon_{11}{}^{(0)} + B_{11}k_{11} + D_{11}k_{11}{}^{(0)} \qquad (8.1)$$

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Volume 4,Issue 1,2016
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$$M_{xx} = B_{11}\varepsilon_{11}{}^{(0)} + D_{11}k_{11} + E_{11}k_{11}{}^{(0)} \quad (8.2)$$
$$L_{xx} = D_{11}\varepsilon_{11}{}^{(0)} + E_{11}k_{11} + F_{11}k_{11}{}^{(0)} \quad (8.3)$$
$$Q_{13} = A_{55}\gamma_{13}{}^{(0)} + B_{55}\gamma_{13}{}^{(1)} \quad (8.4)$$
$$R_{13} = B_{55}\gamma_{13}{}^{(0)} + D_{55}\gamma_{13}{}^{(1)} \quad (8.5)$$

Where

$$\{A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}\} = \int_{-h/2}^{h/2} C_{ij}\{1, z, z^2, z^3, z^4\}dz$$

BOUNDARY CONDITIONS

For a sandwich beam with simply supported condition at x=0, L as follow

$$w = 0 \quad (9)$$

METHOD OF SOLUTION

The Navier method is used for static analysis in the simply supported sandwich beam. Field can be assumed

$$\begin{Bmatrix} u_0(x,z,t) \\ \Phi_1(x,z,t) \\ \Phi_2(x,z,t) \\ w_0(x,z,t) \end{Bmatrix} = \begin{Bmatrix} U\cos\alpha x \\ \Psi_1\cos\alpha x \\ \Psi_2\cos\alpha x \\ W\sin\alpha x \end{Bmatrix} e^{i\omega t} \quad (10)$$

Where

$$\alpha = \frac{m\lambda}{L}$$

By substituting equation (10) into equations (6), four differential equations can be obtained as

$$\begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{21} & s_{22} & s_{23} & s_{24} \\ s_{31} & s_{32} & s_{33} & s_{34} \\ s_{41} & s_{42} & s_{43} & s_{44} \end{bmatrix} \begin{Bmatrix} U \\ \Psi_1 \\ \Psi_2 \\ W \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ p \end{Bmatrix} \quad (11)$$

Where

$s_{11} = \alpha^2 A_{11}, \quad s_{12} = \alpha^2 B_{11}$
$s_{13} = \alpha^2 D_{11}, \quad s_{14} = 0$
$s_{21} = \alpha^2 B_{11}, \quad s_{22} = \alpha^2 D_{11} - A_{55}$
$s_{23} = \alpha^2 E_{11} - 2B_{55}, \quad s_{24} = -\alpha A_{55}$
$s_{31} = \alpha^2 D_{11}, \quad s_{32} = \alpha^2 E_{11} - 2B_{55}$
$s_{33} = \alpha^2 F_{11} - 4D_{55}, \quad s_{34} = -2\alpha B_{55}$
$s_{41} = 0, \quad s_{42} = \alpha A_{55}, \quad s_{43} = 2\alpha B_{55}$
$s_{44} = \alpha^2 A_{55} + K_w w$
$m_{11} = I_0, \quad m_{12} = I_1, \quad m_{13} = I_1, \quad m_{14} = 0$
$m_{21} = I_1, \quad m_{22} = I_2, \quad m_{23} = I_3, \quad m_{24} = 0$
$m_{31} = I_2, \quad m_{32} = I_3, \quad m_{33} = I_4, \quad m_{34} = 0$
$m_{41} = 0, \quad m_{42} = 0, \quad m_{43} = 0, \quad m_{44} = I_0$

NUMERICAL RESULTS AND DISCUSSION

The material properties of metal, ceramic and FGM layers are as the following

$\nu = 0.3, \quad E_{metal} = E_{aluminum} = 70$ Gpa
$E_{ceramic} = E_{alumina} = 380$ Gpa

The effects of volume fraction index (n) on the transverse displacement ($\overline{w}$) of simply supported FGM beam is investigated, and the non-dimensional transverse displacement ($\overline{w}$) obtained using the second order shear deformation beam theory (SBT) for L/h=5 is compared with third order shear deformation beam theory (TBT) results [19] in Table 1. As can be seen the results of first order shear deformation beam theory is in good agreement with the third order shear deformation beam theory results.

Tab.1. Non-dimensional transverse displacement ($\overline{w}$) of simply supported beam.

n	L/h	Present	Ref. [19]
0.5	5	4.7968	4.8292
1	5	6.2047	6.2599
2	5	7.9735	8.0602

In fig. 2, the effect of material distribution (the power-law exponent) and beam theories on the maximum the vertical displacements of the sandwich beam with FG core is shown for different length to thickness ratio (L/h) for Winkler spring constant k_w=1 Gpa. It is seen from Fig. 2 that increase in the material power law index (n) causes decrease in the vertical deflections for all values of the length to thickness ratio (L/h).

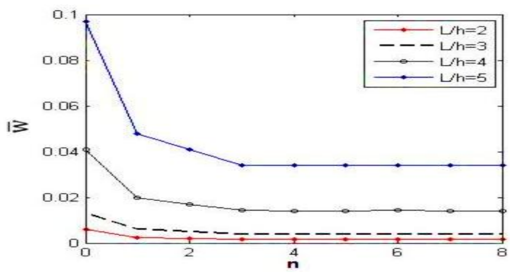

Fig. 2: The effect of material distribution (n) on the maximum vertical displacements of the sandwich beam with FG core.

Table 2 contains non dimensional deflection of FG beams under uniform load P for different values of power law index k and L/h=5. The obtained results are compared with various shear deformation beam theories (SBT, FBT).

Tab.2. Non-dimensional transverse displacement ($\overline{w}$) of simply supported beam.

n	L/h	SBT	FBT
0.5	5	4.7968	4.7384
1	5	6.2047	6.1487
2	5	7.9735	8.0602

Fig. 3 shows that the effect of Winkler spring constant (k_w) on the maximum the vertical displacements of the sandwich beam with FG core for different length to thickness ratio (L/h) for the power-law exponent n=0.5. With increase in the Winkler parameter (k_w), the vertical displacements of the sandwich beam with FG core decreases.

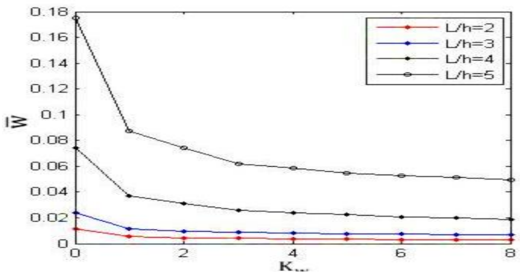

Fig. 3: The effect of Winkler parameter (K_w) on the maximum vertical displacements of the sandwich beam with FG core.

Conclusion

Static analysis of sandwich beams with FGM core resting on Winkler foundation are investigated using second order shear deformation beam theory (SBT). Material properties of the beam change in the thickness direction according to power law distributions. The governing equations are derived using the principal of the minimum total potential energy. The Navier type solution is used for simply supported boundary conditions and exact formulas are proposed for the bending. Numerical results show that the material distribution and foundation parameter play a major role on the response of the bending of the sandwich beam with FGM core.

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