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Buckling Analysis of Rectangular Sandwich Plates with FGM Core Using SSDT

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ABSTRACT

In this paper, an analytical approach for buckling analysis of rectangular sandwich plates with functionally graded (FG) core is presented. Material properties of the FGM layer are assumed to vary in the thickness direction according to a power law distribution in terms of the volume fractions of the constituents. The equilibrium equations are derived according to the second order shear deformation theory (SSDT). Using Navier's type solution these equations are solved for the rectangular sandwich plate with simply supported condition. The excellent accuracy of the present analytical solution is confirmed by making some comparisons of the present results with those available in the literature. Finally the effects of side to thickness ratio, grading index and aspect ratio on the critical buckling load of rectangular sandwich plates with FG core are investigated.

KEYWORD

Beam, Buckling, Functionally Graded, SSDT

INTRODUCTION

Functionally graded materials (FGMs) are microscopically inhomogeneous composite materials, in which the mechanical properties vary smoothly and continuously from one surface to the other. This is achieved by a continuous change in composition of the FGMs. Chen and Liew [1] studied the buckling of FG rectangular plates subjected to nonlinearly distributed in-plane edge loads. They used the mesh free method for their analysis. Ni et al. [2] investigated the buckling analysis of laminated composite plates with arbitrary edge supports. They used the higher-order shear deformation plate theory for buckling analysis of a rectangular plate using the Rayleigh– Ritz method. Shufrin and Eisenberger [3] used the Kantorovich method for stability and vibration analysis of plates. Based on the first- and higher-order shear deformation plate theories, they obtained the frequencies and critical buckling loads for an isotropic rectangular plate. Hosseini-Hashemi et al. [4] have developed the closed form solutions in analytical form to study the buckling behavior of in-plane loaded isotropic rectangular FG plates without any use of approximation for different boundary conditions using the Mindlin plate theory. Saidi et al. [5] employed the unconstrained third-order shear deformation theory to analyze the axisymmetric bending and buckling of FG solid circular plates in which the bending-stretching coupling exists. Oyekoya et al. [6] developed Mindlin type and Reissner type element for modeling of FG composite plate subjected to buckling and free vibration. Further, they studied the plate for the effect of different fiber distribution cases and the effects of fire distribution on buckling, and free vibration. Ghannadpour et al. [7] applied finite strip method to analyze the buckling behavior of rectangular FG plats under thermal load. The solution was obtained by the minimization of the total potential energy and solving the corresponding eigenvalue problem. Nosier and Reddy [8] investigated the buckling and vibration analyses of symmetric laminated plates. They used first- and third-order shear deformation theories. Javaheri and Eslami [9] studied the buckling of functionally graded (FG) rectangular plates under in-plane compressive loading. The closed-form solution for a simply-supported plate was obtained based on the classical plate theory. Wu et al. [10] presented the postbuckling analysis of FG rectangular plates.

They used the first-order shear deformation plate theory for the buckling analysis of FG rectangular plates subjected to thermal and mechanical loads and arbitrary boundary conditions. They used finite double Chebyshev polynomial to obtain the postbuckling responses. Mohammadi et al. [11] presented the buckling analysis of thin FG rectangular plates with Levy boundary conditions. They assumed that the material properties vary through the thickness and proposed an analytical method for decoupling the stability equations. The effects of boundary conditions, material properties, and aspect ratios on the critical buckling load were considered in their study. Xiang et al. [12] used a n-order shear

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deformation theory for free vibration of FG and composite sandwich plates. Natarajan and Manickam [13] proposed an accurate theory for bending and vibration of FG sandwich plates in which two common types of FG sandwich plates were considered. Zhang and Zhou [14] used the classical plate theory (CPT) based on neutral surface to study the bending, buckling, and free vibration responses of FG plates. The CPT based on neutral surface was also adopted by Bodaghi and Saidi. [15] to study the buckling of FG plates under nonlinearly varying in-plane loads resting on elastic foundation. Since the CPT ignores the transverse shear deformation effects, their reported results are limited to thin plates. To account for the transverse shear deformation effects, Prakash et al. [16] employed the first-order shear deformation theory (FSDT) based on neutral surface to investigate the influence of neutral surface position on the nonlinear stability behavior of FG plates.

FUNCTIONALLY GRADED MATERIALS

There are some models in the literature that express the variation of material properties in FGMs. The most commonly used of these models is the power law distribution of the volume fraction. According to this model, the material property gradation through the thickness of plate is assumed to be the following form:

$$E = E(z) = (E_c - E_m)\left(\frac{z}{h_f} + \frac{1}{2}\right)^n + E_m \qquad (1)$$

Here E denote the modulus of elasticity of FGM layer, while these parameters come with subscript m or c represent the material properties for metal and ceramic respectively. For FGM layer, the thickness coordinate variable is presented by z while, $h_m \le z \le h_f + h_m$ where h_f is the total thickness of the FGM layer as shown Fig. 1.

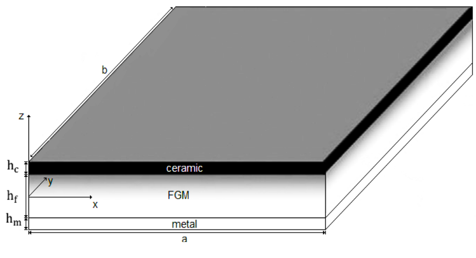

Fig. 1. Geometry and coordinates of laminated plate.

THEORY AND FORMULATIONS

Consider a sandwich plate of length a, width b, thickness h, as shown in Fig. 1. The material in top layer and in bottom layer is ceramic and metal respectively.

DISPLACEMENT FIELD AND STRAINS

According to the coordinate system (x, y, z) shown in Fig. 1, based on second order shear deformation theory (SSDT), the axial and the transverse displacement field are expressed as

$$U_x = u + z\Phi_1 + z^2\Phi_2 \qquad (2.1)$$
$$U_y = v + z\Psi_1 + z^2\Psi_2 \qquad (2.2)$$
$$U_z = w \qquad (2.3)$$

Where (U_x, U_y, U_z) denote the displacement components in the (x, y, z) directions, respectively. All displacement components $(u,\ v,\ w, \Phi_1, \Phi_2, \Psi_1, \Psi_1)$ are functions of position (x, y). The strain-displacement equations of the linear strain are given by

$$\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \gamma_{12}\end{bmatrix} = \begin{bmatrix}\varepsilon^0{}_{11}\\ \varepsilon^0{}_{22}\\ \varepsilon^0{}_{12}\end{bmatrix} + z\begin{bmatrix}k_{11}\\ k_{22}\\ k_{12}\end{bmatrix} + z^2\begin{bmatrix}k^0{}_{11}\\ k^0{}_{22}\\ k^0{}_{12}\end{bmatrix} \qquad (3.1)$$

$$\begin{bmatrix}\gamma_{23}\\ \gamma_{13}\end{bmatrix} = \begin{bmatrix}\gamma^0{}_{23}\\ \gamma^0{}_{13}\end{bmatrix} + z\begin{bmatrix}\gamma^1{}_{23}\\ \gamma^1{}_{13}\end{bmatrix} \qquad (3.2)$$

Where

$$\varepsilon^0{}_{11} = \frac{\partial u}{\partial x}, \quad k_{11} = \frac{\partial \varphi_1}{\partial x}, \quad k^0{}_{11} = \frac{\partial \varphi_2}{\partial x},$$
$$\varepsilon^0{}_{22} = \frac{\partial v}{\partial y}, \quad k_{22} = \frac{\partial \Psi_1}{\partial y}, \quad k^0{}_{22} = \frac{\partial \Psi_2}{\partial y}$$
$$\varepsilon^0{}_{12} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \quad k_{12} = \frac{\partial \varphi_1}{\partial y} + \frac{\partial \Psi_1}{\partial x},$$
$$k^0{}_{12} = \frac{\partial \varphi_2}{\partial y} + \frac{\partial \Psi_2}{\partial x}$$
$$\gamma^0{}_{23} = \Psi_1 + \frac{\partial w}{\partial y}, \quad \gamma^0{}_{13} = \varphi_1 + \frac{\partial w}{\partial x},$$
$$\gamma^1{}_{23} = 2\Psi_2, \quad \gamma^1{}_{13} = 2\varphi_2$$

STRESS-STRAIN RELATIONS

The stress-strain relations are given by

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{23}\\ \sigma_{13}\\ \sigma_{12}\end{bmatrix} = \begin{bmatrix}c_{11} & c_{12} & 0 & 0 & 0\\ c_{12} & c_{22} & 0 & 0 & 0\\ 0 & 0 & c_{44} & 0 & 0\\ 0 & 0 & 0 & c_{55} & 0\\ 0 & 0 & 0 & 0 & c_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{23}\\ \varepsilon_{13}\\ \varepsilon_{12}\end{bmatrix} \qquad (4)$$

Where

$$c_{11} = c_{22} = \frac{E}{(1-\nu^2)}, \quad c_{12} = \nu c_{11}$$
$$c_{44} = c_{55} = c_{66} = \frac{E}{2(1+\nu)}$$

EQUATIONS OF MOTION

The Hamilton's principle for an elastic body can be represented

$$0 = \int_0^T (\delta u + \delta v - \delta k)dt \qquad (5)$$

Hamilton's principle, equation (5), by considering SSDT, equations (2), yields the complete form of the equilibrium equations:

$$\frac{\partial N_{11}}{\partial x}+\frac{\partial N_{12}}{\partial y}=I_0\ddot{u}+I_2\ddot{\varphi}_2+I_1\ddot{\varphi}_1 \tag{6.1}$$

$$\frac{\partial N_{12}}{\partial x}+\frac{\partial N_{22}}{\partial y}=I_0\ddot{v}+I_2\ddot{\Psi}_2+I_1\ddot{\Psi}_1 \tag{6.2}$$

$$\frac{\partial Q_{13}}{\partial x}+\frac{\partial Q_{23}}{\partial y}+\overline{N}=I_0\ddot{w} \tag{6.3}$$

$$\frac{\partial M_{11}}{\partial x}+\frac{\partial M_{12}}{\partial y}-Q_{13}=I_2\ddot{\varphi}_1+I_1\ddot{u}+I_3\ddot{\varphi}_2 \tag{6.4}$$

$$\frac{\partial M_{12}}{\partial x}+\frac{\partial M_{22}}{\partial y}-Q_{23}=I_2\ddot{\Psi}_1+I_1\ddot{v}+I_3\ddot{\Psi}_2 \tag{6.5}$$

$$\frac{\partial L_{11}}{\partial x}+\frac{\partial L_{12}}{\partial y}-2R_{13}=I_2\ddot{u}+I_4\ddot{\varphi}_2+I_3\ddot{\varphi}_1 \tag{6.6}$$

$$\frac{\partial L_{12}}{\partial x}+\frac{\partial L_{22}}{\partial y}-2R_{23}=I_2\ddot{v}+I_4\ddot{\Psi}_2+I_3\ddot{\Psi}_1 \tag{6.7}$$

Where N, M, R, L and Q are the stress resultants. These parameters can be present by

$$\begin{bmatrix}N_{11}\\N_{22}\\N_{12}\end{bmatrix}=\int_{-h/2}^{h/2}\begin{bmatrix}\sigma_{11}\\\sigma_{22}\\\sigma_{12}\end{bmatrix}dz \tag{7.1}$$

$$\begin{bmatrix}M_{11}\\M_{22}\\M_{12}\end{bmatrix}=\int_{-h/2}^{h/2}\begin{bmatrix}\sigma_{11}\\\sigma_{22}\\\sigma_{12}\end{bmatrix}zdz \tag{7.2}$$

$$\begin{bmatrix}L_{11}\\L_{22}\\L_{12}\end{bmatrix}=\int_{-h/2}^{h/2}\begin{bmatrix}\sigma_{11}\\\sigma_{22}\\\sigma_{12}\end{bmatrix}z^2dz \tag{7.3}$$

$$\begin{bmatrix}Q_{13}\\Q_{23}\end{bmatrix}=\int_{-h/2}^{h/2}\begin{bmatrix}\sigma_{13}\\\sigma_{23}\end{bmatrix}dz \tag{7.4}$$

$$\begin{bmatrix}R_{13}\\R_{23}\end{bmatrix}=\int_{-h/2}^{h/2}\begin{bmatrix}\sigma_{13}\\\sigma_{23}\end{bmatrix}Zdz \tag{7.5}$$

And

$$\overline{N}=\overline{N}_x\frac{\partial^2 w_0}{\partial x^2}+\overline{N}_y\frac{\partial^2 w_0}{\partial y^2} \tag{8}$$

BOUNDARY CONDITIONS

For a rectangular sandwich plate with simply supported condition as follow

$$x=0,a\rightarrow\begin{cases}w(0,y)=0\\w(a,y)=0\end{cases} \tag{9.1}$$

$$y=0,b\rightarrow\begin{cases}w(x,0)=0\\w(x,b)=0\end{cases} \tag{9.2}$$

METHOD OF SOLUTION

The Navier method is used for buckling analysis in the simply supported sandwich plate with FGM core. Field can be assumed

$$u=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}U_{mn}\ \cos\alpha x\sin\beta y \tag{10.1}$$

$$v=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}V_{mn}\ \sin\alpha x\cos\beta y \tag{10.2}$$

$$w=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}W_{mn}\ \sin\alpha x\sin\beta y \tag{10.3}$$

$$\varphi_1=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\Phi_{1_{mn}}\cos\alpha x\sin\beta y \tag{10.4}$$

$$\varphi_2=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\Phi_{2_{mn}}\cos\alpha x\sin\beta y \tag{10.5}$$

$$\Psi_1=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\Phi_{3_{mn}}\cos\alpha x\sin\beta y \tag{10.6}$$

$$\Psi_2=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\Phi_{4_{mn}}\cos\alpha x\sin\beta y \tag{10.7}$$

Where

$$\alpha=\frac{m\pi}{a},\qquad\beta=\frac{n\pi}{b}$$

By substituting equation (10) into equations (6), seven differential equations can be obtained as

$$\{[S]_{7*7}-[L]_{7*7}\}\begin{Bmatrix}U\\V\\W\\\Phi_1\\\Phi_2\\\Phi_3\\\Phi_4\end{Bmatrix}=\begin{Bmatrix}0\\0\\0\\0\\0\\0\\0\end{Bmatrix} \tag{11}$$

Where

$s_{11}=A_{11}\alpha^2+A_{66}\beta^2,\qquad s_{12}=(A_{12}+A_{66})\alpha\beta$
$s_{13}=0,\qquad s_{14}=B_{11}\alpha^2+B_{66}\beta^2$
$s_{15}=D_{11}\alpha^2+D_{66}\beta^2,\qquad s_{16}=(B_{12}+B_{66})\alpha\beta$
$s_{17}=(D_{12}+D_{66})\alpha\beta$
$s_{21}=(A_{12}+A_{66})\alpha\beta,\qquad s_{22}=A_{66}\alpha^2+A_{22}\beta^2$
$s_{23}=0,\qquad s_{24}=(B_{12}+B_{66})\alpha\beta$
$s_{25}=(D_{12}+D_{66})\alpha\beta,\qquad s_{26}=B_{66}\alpha^2+B_{22}\beta^2$
$s_{27}=D_{66}\alpha^2+D_{22}\beta^2$
$s_{31}=0,\qquad s_{32}=0,\qquad s_{33}=A_{55}\alpha^2+A_{44}\beta^2$

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$s_{34} = A_{55}\alpha$
$s_{35} = 2B_{55}\alpha, \quad s_{36} = A_{44}\beta, \quad s_{37} = 2B_{44}\beta$
$s_{41} = B_{11}\alpha^2 + B_{66}\beta^2, \quad s_{42} = (B_{12} + B_{66})\alpha\beta$
$s_{43} = A_{55}\alpha, \quad s_{46} = (D_{12} + D_{66})\alpha\beta$
$s_{44} = D_{11}\alpha^2 + D_{66}\beta^2 + A_{55}$
$s_{45} = E_{11}\alpha^2 + E_{66}\beta^2 + 2B_{55}$
$s_{47} = (E_{12} + E_{66})\alpha\beta$
$s_{51} = D_{11}\alpha^2 + D_{66}\beta^2, \quad s_{52} = (D_{12} + D_{66})\alpha\beta$
$s_{53} = 2B_{55}\alpha, \quad s_{56} = (E_{12} + E_{66})\alpha\beta$
$s_{54} = E_{11}\alpha^2 + E_{66}\beta^2 + 2B_{55}$
$s_{55} = F_{11}\alpha^2 + F_{66}\beta^2 + 4D_{55}$
$s_{57} = (F_{12} + F_{66})\alpha\beta$
$s_{61} = (B_{12} + B_{66})\alpha\beta, \quad s_{62} = B_{66}\alpha^2 + B_{22}\beta^2$
$s_{63} = A_{44}\beta, \quad s_{64} = (D_{12} + D_{66})\alpha\beta$
$s_{65} = (E_{12} + E_{66})\alpha\beta$
$s_{66} = D_{66}\alpha^2 + D_{22}\beta^2 + A_{44}$
$s_{67} = E_{66}\alpha^2 + E_{22}\beta^2 + 2B_{44}$
$s_{71} = (D_{12} + D_{66})\alpha\beta, \quad s_{72} = D_{66}\alpha^2 + D_{22}\beta^2$
$s_{73} = 2B_{44}\beta, \quad s_{74} = (E_{12} + E_{66})\alpha\beta$
$s_{75} = (F_{12} + F_{66})\alpha\beta$
$s_{76} = E_{66}\alpha^2 + E_{22}\beta^2 + 2B_{44}$
$s_{77} = F_{66}\alpha^2 + F_{22}\beta^2 + 4D_{44}$
$L_{33} = \lambda_1 N_{cr}\alpha^2 + \lambda_2 N_{cr}\beta^2$

And

$$A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij} = \int_{-h/2}^{h/2} C_{ij}(1, z, z^2, z^3, z^4)dz$$

NUMERICAL RESULTS AND DISCUSSION

The material properties of metal, ceramic and FGM layers are as the following

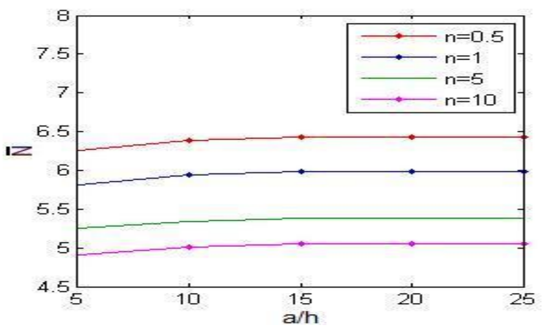
$\nu = 0.3, \ E_{metal} = E_{aluminum} = 70$ Gpa
$E_{ceramic} = E_{alumina} = 380$ Gpa

To validate the accuracy of the present second order shear deformation theory (SSDT) in predicting the critical buckling load of rectangular FGM plate subjected to in-plane loading (biaxial compression and tension: $\lambda_1 = -1, \lambda_2 = 1$) is presented and discussed. The results predicted by present theory are compared with Thai and Chois [17] results and good agreement between the results can be observed. The results are presented in Table 1. It can also be seen that, under biaxial compression and tension, the critical buckling load decreases with the increase of power-law index value while it increases with the increase of side to thickness ratio.

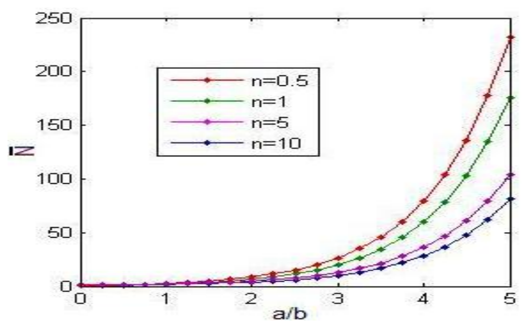
Tab. 1.Comparison non dimensional critical buckling load ($\bar{N}$) of FGM plate subject to biaxial compression and tension ($\lambda_1 = -1, \lambda_2 = 1$).

n	a/h=5		a/h=10	
	Present	Ref. [17]	Present	Ref. [17]
0	9.8564	8.9604	10.8612	9.8738
0.5	6.4878	5.8980	7.0702	6.4275
1	5.0106	4.5551	5.4429	4.9481
5	3.1510	2.8646	3.5441	3.2219
10	2.8179	2.5617	3.2114	2.9195

The effect of side-to-thickness ratio (a/h), and aspect ratio (a/b) on non dimensional critical buckling load $\left(\bar{N} = N_{cr}\, {a^2}/{E_c h^3}\right)$ for simply supported rectangular sandwich plate with FG core is investigated. Figs. 2a, b represent the variation of non dimensional critical buckling load with side-to-thickness ratio (a/h), and aspect ratio (a/b), respectively, under uniaxial compression. From Fig. 2a, it can be observed that critical buckling load decreases with the increase of power-law index values. The increase of aspect ratio (a/b) increases the critical buckling load due to increase of stiffness of the plate as shown in Fig. 2b.



(a)



(b)

Fig. 2: Effect of side-to-thickness ratio (a/h) and aspect ratios (a/b) on non dimensional critical buckling load ($\bar{N}$) under uniaxial compression for a simply supported rectangular sandwich plate with FG core .

The effect of side-to-thickness ratio (a/h) , aspect ratio (a/b), and power law index values on non dimensional critical buckling load $\left(\bar{N} = N_{cr}\, {a^2}/{E_c h^3}\right)$ for a simply supported sandwich plate with FG core under biaxial compression is shown in Figs 3a, and 3b. It can be observed that critical buckling load decreases with the increase of power-law index values.

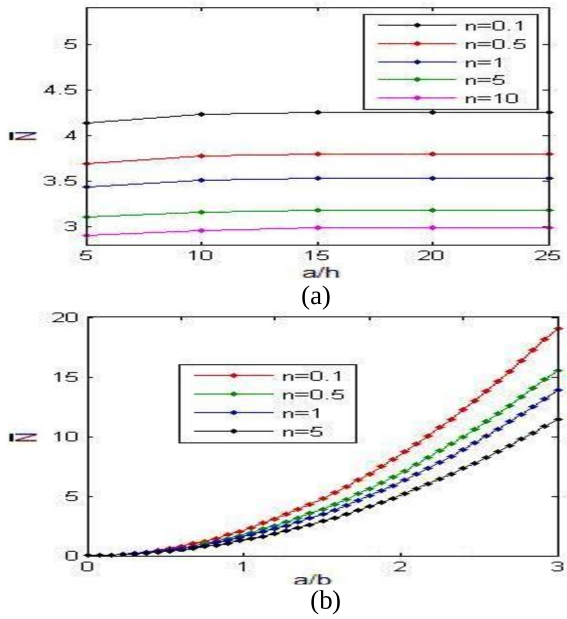


Fig. 3: Effect of side-to-thickness ratio (a/h) and aspect ratios (a/b) on non dimensional critical buckling load ($\bar{N}$) under biaxial compression for a simply supported rectangular sandwich plate with FG core .

CONCLUSION

In the present article, the buckling response of rectangular sandwich plates with FGM core is studied using the second order shear deformation theory (SSDT). The governing equations are derived using the principal of the minimum total potential energy. The material properties are assumed to vary in an power law in thickness direction with the Poisson ratio to be constant.The present formulation was compared with the refined theory developed by Thai and Chois [17] and proved very accurate for the buckling problem. The critical buckling loads have been presented in tables and figures versus variation of different parameters. The critical buckling loads decrease as the power of FGM increases. Increasing the aspect ratio (a/b) ratio increases the critical buckling loads.

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