

Flow over a Broad-Crested Weir in Subcritical Flow Conditions, Physical Syudy

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ABSTRACT

Basic experiments were conducted on rectangular broad-crested weirs with different geometry. It was found that the discharge coefficient of a rectangular broad-crested weir is related to upstream total head above the crest, length of weir and Channel breadth. Multiple regression analysis equations based on the dimensional analysis concept were developed for computing the discharge coefficient of a rectangular broad-crested weirs and discharge coefficient equation was used for computing the discharge over the broad-crested weirs. Good agreements between the measured values and the values computed from the predictive equation are obtained. Therefore, a reliable equation for calculating the discharge coefficient of rectangular broad-crested weirs in subcritical flow conditions is presented.

KEYWORD

Rectangular broad-crested weir, Discharge coefficient, Dimensional analysis, Subcritical flow conditions.

INTRODUCTION

Weirs are a small overflow-type dams commonly used to raise the level of a river or stream and cause a large change of water level behind them. The use of portable instrument like kinds of weirs, flumes, floats, and volumetric tank are common. Discharges measured range from a trickle in ditch to a flood on the Amazon. Many researchers have studied the head discharge relations for flows over sharp-crested weirs and broad-crested weirs with

a simple cross section shape, such as rectangular, triangular, trapezoidal, truncated triangular, and others [10, 24]. Some useful empirical discharge equations for these weirs have been proposed. A broad-crested weir is a flat-crested structure with a length L_{crest} large compared to the flow thickness [14, 20]. The crest is termed broad when the flow streamlines are parallel to the crest and the pressure distribution is hydrostatic [14, 19]. When the crest is "broad", the streamlines become parallel to the crest invert and the pressure distribution is hydrostatic. The discharge above the weir equals:

$$Q = C_d \sqrt{g \left(\frac{2}{3} h_1 \right)^3} . B \quad (1)$$

Where Q is the discharge, B is the channel breadth, g is the gravity acceleration, H_1 is the upstream total head above the crest (Fig. 1), and C_d is the dimensionless discharge coefficient. C_d is unity for an ideal fluid flow above the broad-crest.

Study of broad-crested weirs has attracted the attention of many investigators [1]. Musterle [22] and Montes [19] performed experiments on broad crested weirs. Woodburn [32] showed that the discharge coefficient increases up to 8% if the upstream corner of the weirs is curved [1]. Chow [7] developed a relationship for discharge coefficient, using momentum theorem. Ippen [16], using the Bernoulli and boundary layer equations, developed a relationship for discharge coefficient as a function of boundary layer thickness [1]. Lewith [18] introduced the discharge coefficient as a function of the head of water over the weir, the length and width of the weir, and the flow viscosity [31]. Henderson [15] developed an equation to determine the discharge coefficient for round corner weirs with critical flow condition, assuming $Y_b = 0.715 Y_c$ (Y_c is the critical depth and Y_b is the depth of water at the edge of the weir). In these studies it is also mentioned that separation of flow can be eliminated, if a broad crested weir with upstream round corner is used. Also the critical depth will move downstream, resulting in a subcritical flow dominating all over the weir.

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Belanger [4, 5] analyzed theoretically the overflow and he derived Eq. (1) for the ideal case ($C_d = 1$). Successful physical studies included Bazin [3], Woodburn [32], Tison [30] and Serre [28] Hall [13] and Isaacs [3] studied the effects of developing boundary layer on the overflow. Ramamurthy et al. [23] investigated systematically the discharge characteristics of round-edged and square-edged weirs and Sargison and Percy [26] showed the influence of the weir inflow design on the bottom pressure distributions and discharge coefficient. Based on Sarker and Rhodes [27] works, measurements of the free surface profile over a laboratory scale, rectangular broad-crested weir were performed and were compared with numerical calculations using commercial software. For the given flow rate, the prediction of the upstream water depth was excellent and the rapidly varied flow profile over the crest was reproduced quite well. In the supercritical flow downstream, a stationary wave profile was observed and reproduced in form by the calculations. Gonzalez and Chanson [11] conducted experiments in a near full-scale broad-crested weir. Detailed velocity and pressure measurements were performed for two configurations. The results showed the rapid flow distribution at the upstream end of the weir, while an overhanging crest design may affect the flow field. Clemmens et al [8], studied RBC (Replogle-Bos-Clemmens) broad-crested weirs for circular sewers and pipes. The modified RBC broad-crested weir has many advantages over related open channel flow devices. These include high accuracy and reliability for a wide variety of shapes, low head-loss requirements which are predictable, and relatively simple inexpensive construction. Based on their investigations, theoretical equations were presented for ideal flow from which approximate ratings can be obtained to within a reasonable accuracy with an empirical discharge coefficient, however, a mathematical model is available which accurately predicts these ratings by directly accounting for the effects of friction. The ratings for a wide variety of shapes and sizes of these weirs were computed with the model and fit to an empirical equation. Design examples are given which show how to select the flume dimensions for maintaining free flowing conditions (modular flow) and for minimizing sediment deposition.

The purpose of this study is investigation of discharge coefficient (C_d) in rectangular broad-crested weirs. In addition comparisons of present results with other researchers study have been carried out.

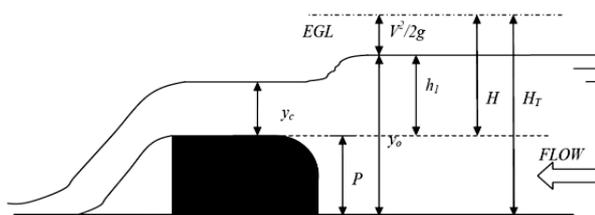


Fig. 1. Definition sketch of a broad-crested weir

New experiments were conducted at the laboratory of Islamic Azad University of East Tehran branch, Iran, in a research flume that was made of glass with a cross section 0.30 m wide, 0.50 m deep and 4.8 m long (Fig. 2). Water was supplied from a large 1.5 m deep feeding basin leading to a sidewall convergent enabling a very smooth and wave less inflow. The weirs are consisted of a 0.038 m height, 0.30 m width, with an upstream rounded corner (0.0134 m radius) and 0.187 m and 0.336 m long flat horizontal crest respectively (Table 1). A pump controlled with an adjustable frequency AC motor drive delivered the flow rate, enabling an accurate discharge adjustment in a closed-circuit system. Clear-water flow depths were measured on the channel centerline with a point gauge and using photographs through the sidewalls. The accuracy of point gauge and photographic data yielded the same results within 1 mm.



(a)



(b)

Fig.2. Experimental Set-Up (a) Laboratory flume. (b) broad-crested weir in action.

EXPERIMENTAL LAYOUT

Table 1- Experimental investigations of horizontal broad-crested weirs

Experiment	L	Δz	B	Q	H ₁
(1)	(2)	(3)	(4)	(5)	(6)
Geometry 1	0.187	0.38	0.30	0.00133 to 0.006	0.0234 to 0.0508
Geometry 2	0.336	0.38	0.30	0.00133 to 0.006	0.0252 to 0.0512

Table 2- Equations used by other researcher for C_d in broad-crested weir

Experiment	Equation	Condition
GOVINDA RAO & MURALIDHAR [12]	$C_d = 0.521 + 0.028(\frac{H_1}{L})$	$0.1 < (H_1 / L) < 0.35$
AZIMI & RAJARATNAM [2]	$C_d = 0.9 + 0.147(H_1 / (H_1 + P))$	$0.1 < (H_1 / L) < 0.4$
FELDER & CHANSON [9]	$C_d = 0.92 + 0.153(\frac{H_1}{L})$	$0.02 < H_1/L < 0.3$
SALMASI et al. (2012)	$C_d = 0.612 + \frac{H_1}{L}$	$H_1/L \leq 0.27$

RESULTS AND DISCUSSION

The discharge coefficient (C_d) of rectangular broad-crested weir can be written as a function of the width of the channel (B), total energy head upstream of the weir (h₁), mean flow velocity in the main channel (v), length of broad-crested weir (L), and acceleration due to gravity (g).

$$C_d = f(B, h_1, L, v, g) \tag{2}$$

Dimensional analysis based on Buckingham’s theorem was used to find non-dimensional variables in the present study.

$$C_d = f(\frac{V}{\sqrt{gh_1}}, \frac{h_1}{B}, \frac{h_1}{L}) \tag{3}$$

In which Fr = $\frac{V}{\sqrt{gh_1}}$ is the Froude number in the main channel. Therefore, Eq. (3) reduces to:

$$C_d = f(Fr, \frac{h_1}{B}, \frac{h_1}{L}) \tag{4}$$

The discharge coefficient was computed using the Equation (1). In order to estimate the outflow over a rectangular broad-crested weir, the discharge coefficient in the weir equation needs to be known. Equation (5) was developed for computing the discharge coefficient of a rectangular broad-crested weir from the experimental results. The SPSS mathematical software has been used to consider simultaneous effects of dimensionless parameters on discharge coefficient. The variation of Froude number did not have significant effect on discharge coefficient. Thus, the dimensionless discharge coefficient data was best correlated by:

$$C_d = 6.419(\frac{h_1}{B})^{0.0669} - 10.461(\frac{h_1}{L})^{0.0135} \tag{5}$$

Free surface profiles over rectangular broad-crested weirs for different value of discharge and weir length are shown in Figures 3 and 4. Figure 5 shows the comparison of inlet and

outlet values of rectangular broad-crested weir discharge (Q) for different Geometries used in this study. In Figure 6, head-discharge for various weir lengths is presented. Figure 7 shows the comparison of observed and calculated triangular broad-crested weir discharge coefficient using Equation (5) for all the experimental data. Also, the comparison of observed and calculated discharge coefficient using Equation (5) and equations used by other researchers (Table 2) is shown in Figure 8.

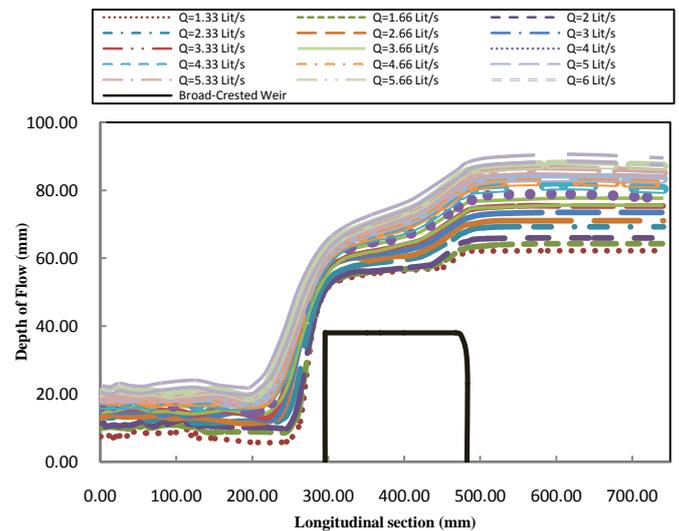


Fig.3. Free surface profiles above a broad-crested weir for L=18.7cm

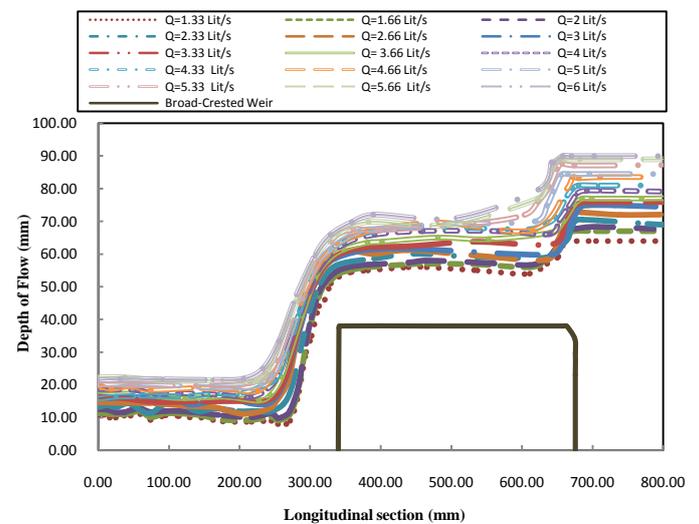


Fig.4. Free surface profiles above a broad-crested weir for L=33.6cm

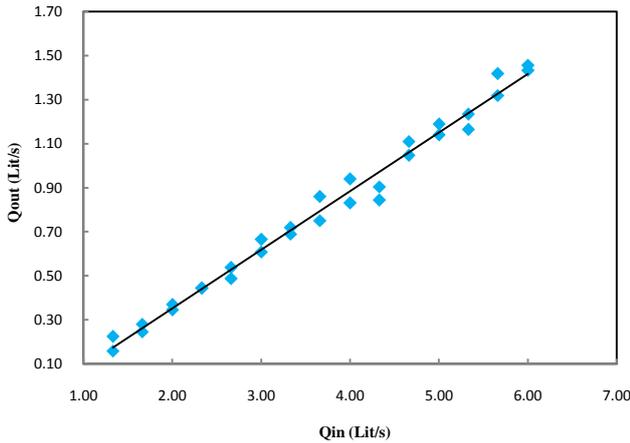


Fig.5. Comparison of inlet and outlet values of broad-crested weir discharge

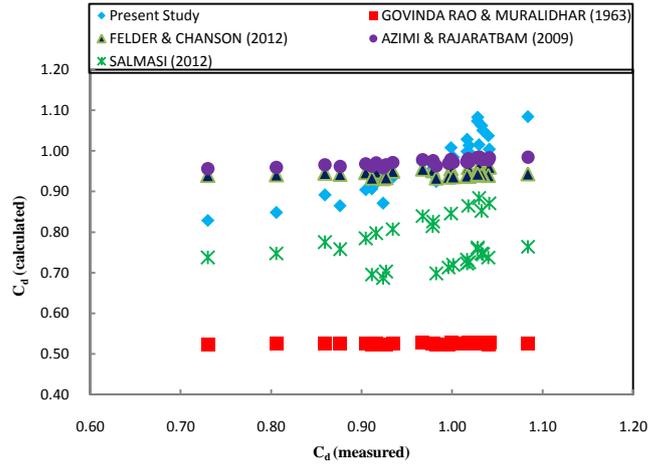


Fig.8. Comparison of measured and calculated discharge coefficient using different equation

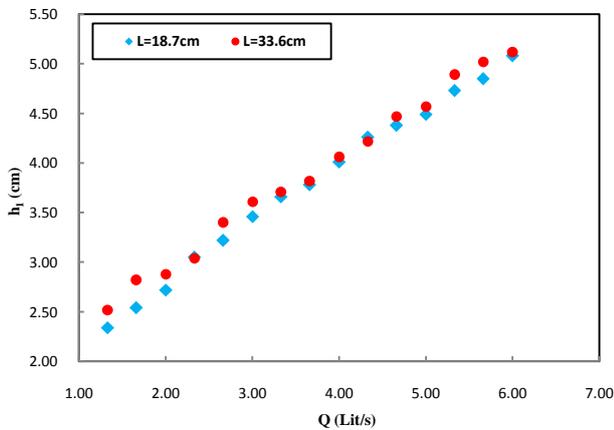


Fig.6. Upstream total head above the weir crest according to different value of discharge

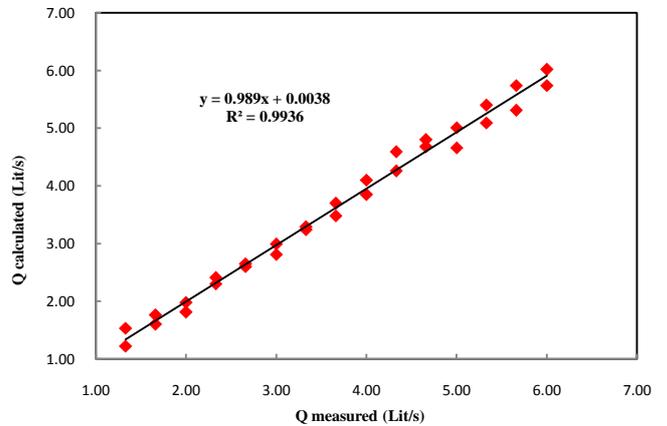


Fig.9. Comparison of measured and calculated discharge of broad-crested weir

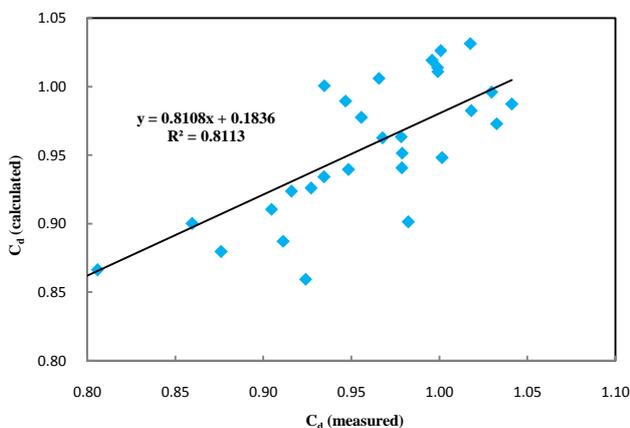


Fig.7. Comparison of measured and calculated discharge coefficient of broad-crested weir

The mean relative error percentage (RE), root mean square errors (RMSE), mean absolute errors (MAE), slope of regression line (k) and coefficient of determination (R^2) statistics are used to evaluate the model accuracies. R^2 is defined as the ratio of the sum of squares explained by a regression model and the “total” sum of squares around the mean. Different types of information about the predictive capabilities of the model are measured through RMSE and MAE. The RMSE sizes the goodness of the fit related to high discharge coefficient values whereas the MAE measures a more balanced perspective of the goodness of the fit at moderate discharge coefficients. The k is constant and it is shown that the present model is very close to the line of perfect agreement. The RE, RMSE and MAE are defined as:

$$RE = \frac{100}{N} \sum_{i=1}^N \left| \frac{Q_{0i} - Q_{ci}}{Q_{0i}} \right| \quad (6)$$

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (Q_{0i} - Q_{ci})^2} \quad (7)$$

$$MAE = \frac{1}{N} \sum_{i=1}^N |Q_{0i} - Q_{ci}| \quad (8)$$

Q_c is the calculated discharge of the broad-crested weir, Q_o is the measured discharge of broad-crested weir, N is the total number of runs and i is the i th run. Table 3 shows the value of RE, MAE and RMSE for present study.

Table 3- Value error of present study

RE	RMSE	MAE
3.043	0.123	0.0987

CONCLUSION

In this study, laboratory measurements were carried out on rectangular broad-crested weir with different geometries located on a straight rectangular main channel to investigate the new equation for discharge coefficient. As a result of dimensional analysis, the results indicate that the dimensionless parameter of h_1/B should not be ignored in equations determining the discharge coefficient of the rectangular broad-crested weir. Multiple regression analysis equations based on the dimensional analysis concept were developed for computing the discharge coefficient of a rectangular broad-crested weir; and discharge coefficient equation was used for computing the discharge over rectangular broad-crested weir. The present results also showed a slight increase in discharge coefficient with increasing head above crest similar to previous studies. The mean relative error percentage (RE), root mean square errors (RMSE), mean absolute errors (MAE) and correlation coefficient (R) values for Eq. (5) are 3.043, 0.123, 0.0987 and 0.9936, respectively. Good agreements between the measured values and the values computed from the predictive equation are obtained. Thus, an accurate equation for the discharge coefficient of the rectangular broad-crested weirs in subcritical flow conditions is introduced.

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NOTATION

B	Channel breadth; m
C_d	Dimensionless discharge coefficient
g	Gravity acceleration; m^2s^{-1}
H	Total head; m
H_1	Upstream total head measured above the crest; m
H_T	Total energy head of upstream flow measured relative to the base of the flume; m
L	Length of broad-crested weir; m
Q	Water discharge; m^3s^{-1}
V	Upstream velocity; ms^{-1}
Fr	Froude number
y_0	Upstream water depth; m
y_c	Critical water depth at control section; m
P	Broad-crested weir height; m

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