Modeling and Prediction of Stock Market Volatility using Markov-Switching Model

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ABSTRACT

In world markets, a variety of volatility can be observed in stock market, which can exert different effects on the economy of a country and results in developing suitable economic policies. In the present article, the stock market volatility was first modelled using the data of Tehran Stock Exchange from June 1992 to January 2012, then Markov-switching model was used to predict stock market volatility in a non-eventual state. The reason for using the model is that there is a possibility of switching or transition between two regimes for the indexes of the model. As a result of this, normal distribution and t distribution as well as GED were observed for errors. In order to predict stock market volatility of Iran, the performance of the Markov-switching model can be way better than other distributions with respect to t error distribution with degrees of freedom between the two regimes. According to transition probabilities, volatility of stock price would hinder the improvement of the situation, as it may cause stock market switch from a situation to a lower one. However, the great tendency for stability of the situation of a regime in proportion to transition to another situation indicate that an accurate planning can hinder the transition to lower situation (recession)

KEYWORDS
Markov-switching, transition probability, volatility, regime transition

INTRODUCTION

Today, Markov-switching models have appealed to many researchers with respect to economy modelling, because structural and non-linear changes and dynamic features of time series can be observed in this model as empirical evidence suggests. In this respect, the dynamic behavior of time series of microeconomics depends on non-linearity with respect to stages of business cycles, and it is mostly used to stabilize and identify the transition behavior of regimes concerning recession and economic boom. Since the pace of change from one situation to another is invariably fast, the model allows us to model the variant behavior which rapidly transmits between both situations.

Markov-switching technique allows us to account for a variety of new questions: can we recognize different regimes with respect to the returns of stock market? How does regime change? With what possibility do regimes undergo transition or stay in the same regime? the indexes of the model were estimated by maximum verification (the studies of Engel and Hamilton, 1990). Economy analyst postulates that the transitions cannot be observed directly, but instead he needs to have some kind of probability inference about when and where the switch and changes may occur, and the inference is drawn based on observed behavior of the series itself. For the idea beyond transition model, as market situations changes, so do contributing factors in volatility.

For instance, the process of fluctuations works differently in recession in proportion to the situation of economic boom. In Markov-switching, the levels of volatility would change between two levels of regime. It is possible to have more than two regimes, but it is assumed that there are only two regimes for matter of simplicity. The indexes of the model would have different values for either regime. Regimes here are called non-observable variables and state variable. Therefore, Markov-switching regime model is introduced as a suitable tool for analyzing and forecasting stock market volatility.

The modelling of stock returns play a leading role in research. The results of the prediction of the model are viewed as a fundamental factor in risk and recession management as well as for financial policy makers. Today, the fact become evident that financial volatility is a kind of random process with a degree of persistence in stock return volatility. The aim of using Markov-switching model is that we can obtain the probability of swirling through the random component of the model which is the state of the same regime and follows a Markov process.

In what follows. Having presented the introduction, in the second part a review of conducted studies will be introduced. In the third part, the theoretical foundation of the study is posited, and the article will introduce the model and its specifications and the method of study in part 4. In the fifth part of the study, the estimate of the model and analysis of findings will be discussed, and finally, in the sixth part, the overall conclusion is presented.
2- Review of Literature

Due to structural failures, macroeconomic variables are conceived as one of the features of time series, for which the implementation of policies or economic plans come up through exogenous shocks or changes in economic regime. The factors can bring about change in the trend of stock market index fluctuations. It is a long time since economists were concerned about the econometric recognition of stock market volatilities. Earlier the studies focused on the indirect recognition of fluctuations in stock market. However, indirect tests for stock market have been often criticized, because the effects of fluctuations in stock prices cannot be easily distinguished from the basic unobservable effects of market. As a result, the direct examination of fluctuations have been developed, so to speak, which examines and identify the cause of some kind of volatility. As an alternative regime completely different from autoregressive process, stock market volatility is viewed by the West as a constant process. They also interpret it as statistically different between two alternative estimators of a particular parameter as a sign of fluctuation. In recent years, many foreign studies have been conducted on Markov-switching models, among which Gojin Chen (2011) deals with Markov’s transition models and forecasting state space as well as comparing stock market index of USA with China.

Zhao and Zeng found out that the trend of monthly rate (i.e. return or monthly run) can be clearly split into 2 states; fluctuation persistence state and fluctuation damping state. The trend of fluctuation expansion and damping in China was described by pursuing switching regime model in order to study the nonlinear relationship of return to turnover. Wilfling and Anasawah treated volatility as unobservable variable, e.g. Wu (9). However, they developed its framework by letting fluctuation switch among alternative regimes. In doing so, they were successful in separating two different periods by bubble processing.

In Iran, Minoo Nazifi Naeeni could forecast and model volatility of market fluctuations using Garch-Markov transition model. Farzin and Shakouri (2009), in their thesis on stock return and inflation in Iran, explored the relationship between inflation rate and stock return of Tehran Stock Exchange using Markov’s vector autoregressive model, Markov’s switching model, and switching forecast. Abunouri and Erfani (2008), in their study entitled “Markov’s switching model and forecasting the probability of liquidity crisis in OPEC member countries”, could estimate some kind of model warning prior to their occurrence. It is hoped that the study sets a foundation for Markov switching modelling in Iran stock market. According to the literature and flexibility of our calculations, the characteristics of two regimes are chosen for the article.

3- Theoretical Foundation

3.1- Markov switching model

Markov-switching model was first introduced by Quandt (1972) and Quandt-Goldfeld (1973). In a non-linear model, it is assumed that the variant behavior on which modelling is conducted is varied in different situations. Nonlinear models are divided into two major classes in terms of the pace of change from one situation to another. For some of the models, change from a situation to another takes place smoothly, such as STAR models and ANN artificial network, while for others the transition occurs fast; Markov-switching model is an instance (Andres and Jonso). The equation taps into several equations in order to account for the behavior of variables in different regimes, in that as equations change in different regimes, it is possible for the model to account for convoluted dynamic patterns. The exquisite feature of Markov-switching is that in the model the mechanism of regime change depends on a situation variable. Therefore, the model is fit for accounting for data that reveal various behavioral pattern in different time ranges. The main state of Markov-switching model was proposed by Hamilton for variable means. The state as well as other states have been widely used in an attempt to explore economic and financial variables. One of the advantages of Markov-switching model compared to other methods is an endogenous split of the relationship between the observations of variables. As a result, Markov-switching method is completely different from models based on structural failure and dummy variables. For models based on a structural failure, years of structural failure are set in time series variables either exogenously or endogenously without regard to probabilities, while in Markov-switching model probabilities are taken into consideration in order to separate time series variables and/or the relationships between two or more regimes, i.e. the probability of a transition from a regime to another regime is calculated. Hence, as for structural failure, such issues are not the case as the possibility of forecasting transition from one situation to another is unknown. Furthermore, in the models based on structural failure, there is no possibility of forecasting structural failure as well as the variations of variables. However, there is a possibility of forecasting the variations of variables in Markov-switching model (Asgharpour). In this study, in order to estimate the model, Markov-switching model was used. The method has the capability of illustrating asymmetrical features of regimes due to its nonlinearity, and it is more suitable than VAR and ARIMA methods.

In Markov-switching model, it is assumed that a regime that occurs at t time is not observable and depends on a St unobservable process. In a model with two regimes, we can easily assume that St takes the mandatory values 1 and 2. The two-regime model can be shown as follows:

\[ y_t = \Theta_{0,2} + \Theta_{1,2} z_t + \varepsilon_t \]  

(1)

Where Y stands for dependent variable and Z stands for a vector of elaborative variables. To complete the model, we need to specify the features of St Procedure. In Markov-switching model, St is taken as first rank Markov process. The assumption points out that St is just contingent upon the regime of prior period, i.e. St-1. In what follows, our model is completed by introducing the probabilities of transition from one situation to another:

\[ P(S_t = j | S_{t-1} = i) = p_{ij} \]

\[ S_t \in (1,\ldots,k) \]
In the above equations, $p_{ij}$ stands for the likelihood of Markov chain move from the state at $t$ time to the state at $t-1$ time. The following conditions must be met for $p_{ij}$:

$$
\sum_{i=1}^{k} p_{ij} = 1
$$

$$
0 \leq p_{ij} \leq 1
$$

$$
\begin{bmatrix}
p_{11} & p_{21} & \cdots & p_{k1} \\
p_{12} & p_{22} & & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
p_{1k} & p_{2k} & \cdots & p_{kk}
\end{bmatrix}
$$

4- Methodology of the study and the illustration of the model

4.1. The illustration of Markov-switching model

In terms of computation, Markov-switching model can be presented as follows:

$$
y_t = \sum_{i=1}^{N_k} \beta_i X_{it} nS + \sum_{j=1}^{N_k} \phi_i, s_i X_{j, t} + \epsilon_t
$$

$$
\epsilon_t \sim P(\Phi, S_i)
$$

$$
y_t = \beta_{1S_i} X_{1, t} + \beta_{2S_i} X_{2, t} + \epsilon_t
$$

$$
\epsilon_t \sim N(0, \sigma_{S_i}^2)
$$

Where:

$S_i$: time state in which $St=1,\ldots,k$ and $K$ are the number of circles.

$\sigma_{S_i}^2$: new variance in $St$ state

$\beta_{iS_i}$: Beta coefficients for explanatory variables in a state where $i$ changes from 1 to 2.

$\epsilon_t$: Residual vectors which follows a special distribution (here normal distribution).

Now, if we change $S$ input as $S=[1\ 1\ 0]$, the model is obtained as follows:

$$
y_t = \beta_{1S_i} X_{1, t} + \beta_{2S_i} X_{2, t} + \epsilon_t
$$

$$
\epsilon_t \sim N(0, \sigma_{S_i}^2)
$$

With these changes in the input $S$, only a secondary factor and model variance can be switched with transition probabilities. If $\beta$ ratio would not change, so it is clear that the first components of $S$ can control dynamic switch in the equation, while the last level control dynamic switches. The model GED (generalized error distribution) can be explained for the residuals $S=[1\ 1\ 1\ 1\ 1]$ as follows:

$$
y_t = \beta_{1S_i} X_{1, t} + \beta_{2S_i} X_{2, t} + \epsilon_t
$$

$$
\epsilon_t = GED(0, \sigma_{S_i}^2, K_i)
$$

In this model, the new parameter $K$ switch between states, and the ratio is part of GED distribution which should not be confused with parameter $K$ (i.e. the number of states in the model).

Therefore, the data input $S$, $K$ and dependent variable $Y_t$ and $X_t$ are related to the estimate of the following models.

$$
State1(S_i = 1)
$$

$$
y_t = \beta_1 + \epsilon_t
$$

$$
\epsilon_t \sim N(0, \sigma_1^2)
$$

$$
State2(S_i = 2)
$$

$$
y_t = \beta_2 + \epsilon_t
$$

$$
\epsilon_t \sim N(0, \sigma_2^2)
$$

$$
P = \begin{pmatrix}
P_{1,1} & P_{2,1} \\
P_{1,2} & P_{2,2}
\end{pmatrix}
$$

In matrix, the transition likelihood of the sum of each column equals one, because it brings in the total likelihood for each state.

5- The results and data analysis

A total of data used in the study include the stock price of stock market open days (i.e. five days a week) from 29/06/1992 to 27/11/2013. In addition, the total price index (TEPIX) was used for stock price. Daily price data were converted to daily returns by the following standard method:

$$
r_t = 100 \times LN \left( \frac{p_t}{P_{t-1}} \right)
$$

Now, the presence of a unit root (non-stationary) was examined for returns using ADF (similar to the PP test result)

<table>
<thead>
<tr>
<th>Test statistic ADF</th>
<th>t Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>-14/16661</td>
</tr>
<tr>
<td>5%</td>
<td>-3/431484</td>
</tr>
<tr>
<td>10%</td>
<td>-2/861926</td>
</tr>
<tr>
<td>15%</td>
<td>-2/567018</td>
</tr>
</tbody>
</table>

Since test statistic is more negative than the critical values at 1%, 5%, 10% level, the null hypothesis about the presence of unit root (non-stationary) in favor of the opposite hypothesis (stationary) is rejected. Therefore, our data is not stationary.

The results of estimating the model using Markov-switching method were stated in what follows:
According to the results of table (2) for Markov-switching model (normal distribution) state, regime 1 with 91% probability hinders the improvement of economic situation and admittance to regime 2, as there is only 9% probability that economy can be moved to a higher regime. And, if it is fed into regime 2, it will stay in the same situation with 88% tendency, and it may return to regime 1 only with 12% probability. Therefore, the tendency to stay in each regime was greater. In conclusion, according to the transition probabilities, the occurrence of fluctuations in stock price can not only hinder the improvement of the situation, it can also switch from one situation to lower situation. However, the high tendency to remain in the situations of the first and second regimes with respect to transition to another situation indicate that we can avoid the transition of economy to lower situations (recession) by planning more accurately.

Table 2: the results of Markov-switching estimate

<table>
<thead>
<tr>
<th>Regime</th>
<th>Normal</th>
<th>t student</th>
<th>GED</th>
</tr>
</thead>
<tbody>
<tr>
<td>likelihood</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regime 2</td>
<td>-1240/6369</td>
<td>-1160/087</td>
<td>-1168/0869</td>
</tr>
<tr>
<td>Regime 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0/1821</td>
<td>0/0001</td>
<td>0/0001</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0/03</td>
<td>0/03</td>
<td>0/02</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>0/79</td>
<td>0/23</td>
<td>0/10</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0/06</td>
<td>0/02</td>
<td>0/20</td>
</tr>
<tr>
<td>stability</td>
<td>8/42</td>
<td>11/3</td>
<td>347/112</td>
</tr>
<tr>
<td>Distribution parameter</td>
<td>K GED 0/04</td>
<td>0/04</td>
<td>0/01</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>44/397</td>
<td>397/18</td>
<td></td>
</tr>
</tbody>
</table>

According to the table, regime 1 is the most stable regime, because if economy is fed into the regime according to t and normal distribution, greater number of periods will remain in the regime than regime, which is not consistent with economic theories because as the theories suggest recession period invariably last shorter. According to the likelihood of the transitions observed, regime is the most stable regime, because the likelihood of transition from the regime to the regime itself is quite high. For example, in normal distribution state, regime 1 with 91% probability hinders the improvement of economic situation and admittance to regime 2, as there is only 9% probability that economy can be moved to a higher regime. And, if it is fed into regime 2, it will stay in the same situation with 88% tendency, and it may return to regime 1 only with 12% probability. Therefore, the tendency to stay in each regime was greater. In conclusion, according to the transition probabilities, the occurrence of fluctuations in stock price can not only hinder the improvement of the situation, it can also switch from one situation to lower situation. However, the high tendency to remain in the situations of the first and second regimes with respect to transition to another situation indicate that we can avoid the transition of economy to lower situations (recession) by planning more accurately.

According to figure (1), it is evident that Markov-switching model with normal error distribution for data of

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In Table (2), almost all of the indexes are significant at 95% significance level. In order to see various regimes, non-conditional variance is calculated for each fluctuation. According to the results of table (2) for Markov-switching models with t error distribution, the non-conditional variance of regime 1 is about 53%, and regime 2 23%, which is about four times regime 1. The finding also states that the process of fluctuations in Iran stock market is described with two different regimes. Additionally, the difference between the variance of either regime suggests that the transition of regime is essential to fluctuation models.

The variance of disruption components was different for both regimes, and the numbers suggest that the first regime (recession) has a lower volatility than the second regime (boom period). In the model, t student error with fixed degree of freedom and without change in regimes, the degree of the freedom of the model index was less than 3. The degree of freedom is related to the model and indicate that the moments exist as long as the second moment (Karadazh, 2008). That is to say, as validating state-dependent parameters, we can model a variety of failures and cuts for data.
the total Tehran Stock Exchange index is unable to detect the switching or transition of regime. Moreover, the smoothed probabilities of regime 1 (recession) and regime 2 (boom) were left apart, and there was less transition between both the regimes. On few points, regime 1 entered the area of regime 2, though there was no transition at all that can be observable. The minor transitions between both regimes are very slight and transient.

Figure 2. The transition diagram of regime for Markov-switching model (GED)

Diagram (2) is a representation of the smoothed probabilities of Markov-switching model via GED error distribution. The diagram as opposed to that of figure (1) indicates that even though transition cannot be identified, regime 1 (recession) moves into the area of regime 2 (boom), making transition between both regimes even for a few days.

Figure 3. The regime transition diagram for Markov-switching model (t distribution)

Diagram (3) represents the smoothed probabilities of fluctuations in Markov-switching model with t error distribution as well as the degree of freedom index fixed between both regimes. The transition between both regimes, one and two, is loud and clear. The model could finely depict the regime transitions for the fluctuations in Tehran Stock Exchange. The crossing points between both regimes and fluctuation regime substitution are due to contributing factors in stock exchange which influence stock market.

6. Conclusion:
The main aim of the present study is to explore the fluctuations of stock market in Iran during 1992-2013. In this respect, the daily data of the total Tehran Stock Exchange index were first rendered into daily returns. Then their reliability were examined by means of the unit root test ADF. After this, in this study, using Markov-switching method, the fluctuations over both regimes were studied, and the probabilities of transition from one situation to another or remaining in the same situation have been examined. According to the results obtained, Markov-switching model with t error distribution and fixed degree of freedom between both regimes can beautifully depict the volatility of Tehran Stock Exchange. The model shows that regime 1 is the most stable regime, because it remains the same for more periods according to t and normal distribution, and the matrix of transition probability indicate that the probability of transition from regime 1 to the regime itself is high. In general, the tendency to remain in the same situation is greater in proportion to transition to upper or lower situations. Therefore, we can hold that the occurrence of stock price volatility can improve the situation (transition to boom state) or transition to recession, but if we are transited to boom or recession state, then the tendency to stay in the same situation is more than transition to other situations.

REFERENCES


